

Prediction Method for Sandstone Permeability Based on Fractal Theory

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Abstract

Permeability is an important parameter that controls the seepage capacity of sandstone layers. The complexity of the pore structure and the strength of heterogeneity in sandstone affect the accuracy of permeability prediction. Fractal dimension can analyze the pore space structure of real rock samples from multiple dimensions and different perspectives and describe the characteristics of the pore structure. This study takes six groups of sandstones as the research objects and comprehensively uses thin section identification, HPMI (high-pressure mercury injection) experiments, and fractal theory to conduct analysis from both qualitative and quantitative dimensions. Based on the fractal theory, a permeability prediction model is constructed, and the model is optimized and corrected to establish a mathematical model suitable for predicting the permeability of sandstone.

Keywords

Linxing area; HPMI; Fractal theory; Permeability prediction model.

1. Introduction

The study of sandstone permeability is of great significance for the waterproofing of geotechnical underground engineering and the exploitation of oil and gas resources. Traditional permeability prediction methods have many limitations, and it is difficult to describe complex pore structure characteristics of reservoirs comprehensively and accurately [1]. HPMI technique can obtain abundant pore structure information, and fractal theory provides an effective way to explain this complex structure quantitatively. Wu et al. [2] combined fractal theory with the T-C model and SDR model, respectively, to build two permeability prediction models based on nuclear magnetic resonance experiments. At present, there are few permeability prediction models based on fractal theory and high pressure mercury injection technology. Based on the fractal theory, the permeability prediction model is constructed through high pressure mercury injection experiment to provide theoretical support for the study of sandstone seepage.

2. Pore Structure Characteristics of Sandstone

The six sandstone samples analyzed in this experiment are buried at depths of 1282.80~1716.93 m. The porosity and permeability of sandstone are tested by a conventional porosity experiment. The porosity ranges from 6.25% to 9.74% with an average value of 7.40%, and the permeability ranges from 0.08 to 0.49 mD with an average value of 0.25 mD. The material composition of sandstone is analyzed by thin section identification of rock. The sample

is mainly quartz, the content is 67%~83%, and the clay content is 5%~13%, indicating that clay and siliceous cementation is strong. The content of feldspar and mica is 0%~6% and 2%~30% respectively, and the pore structure is uneven due to the expansion and contraction of minerals [3, 4].

The characteristics of capillary pressure curves are shown in Figure 1. The trend of mercury injection curves of the six rock samples is basically the same. During the mercury injection stage, the capillary pressure of the rock samples is mostly in the range of 1 MPa to 100 MPa within the gentle rising range of the mercury injection curve. The drainage pressure of the rock samples is measured to be between 0.33 MPa and 1.02 MPa, with an average value of 0.58 MPa. A smaller value of the separation coefficient means that the distribution of pore throat size is closer to the average, and the pore structure is more uniform. The separation coefficient of rock samples ranges from 2.08 to 2.84, with an average value of 2.52. The better the rock's physical characteristics, the lower the P_{50} . The median pressure of the rock sample ranges from 2.08 to 7.77 MPa, and the average value is 4.27 MPa. The rock distortion coefficient ranges from 0.15 to 0.44, with an average value of 0.32. The maximum mercury saturation ranges from 85.12 to 94.75%, with an average value of 90.04%. The mercury removal efficiency of rock samples ranges from 32.55 to 50.13%, with an average value of 37.93%.

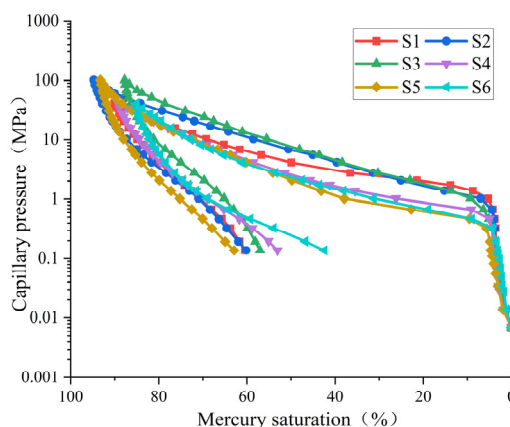


Figure 1. Sandstone capillary pressure curve

Figure 2 shows the pore throat distribution curve of the rock sample, indicating that the pore throat distribution range is 0.0063~1.60 μm , and the curve is a unimodal distribution with a peak range of 0.25~1 μm , showing the characteristics of low porosity, low permeability and complex pore structure. In addition, the reservoir developed a certain large pore throat, but mainly small pore throats.

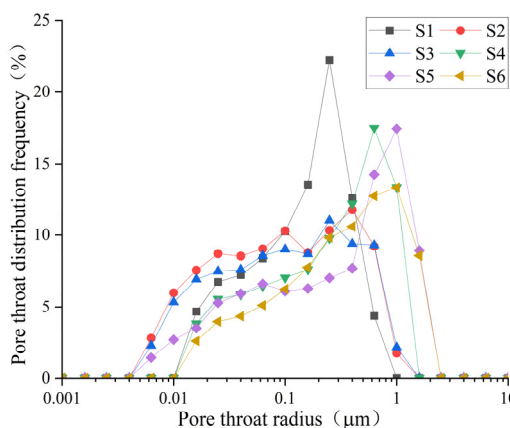


Figure 2. Distribution characteristics of pore throat radius of sandstone

3. Fractal Dimension Calculation

Fractal dimension can be used to quantitatively characterize the complexity and heterogeneity of reservoir pore structure [5]. The fractal dimension D can quantify the heterogeneity of rock. The fractal dimension of sandstone pore structure is between 2.0 and 3.0, and the fractal dimension is close to 3.0, which indicates that the pore surface of sandstone is more irregular and the texture is rough. The fractal dimension is close to 2.0, indicating that the pore surface of sandstone is relatively smooth [6, 7].

The fractal geometric formula describing the distribution of reservoir pore size is as follows:

$$S_w = \left(\frac{r}{r_{\max}}\right)^{3-D} \quad (1)$$

Where S_w is the wetting phase saturation in the reservoir; r_{\max} is the maximum pore radius. Considering the capillary model and the negative correlation between capillary pressure P_c and pore diameter r , the fractal geometric formula of the reservoir capillary pressure curve is as follows:

$$S = \frac{P_c}{P_{\min}}^{D-3} \quad (2)$$

Where P_{\min} is the inlet capillary pressure and the capillary pressure corresponding to the maximum pore diameter r_{\max} in the reservoir; S_w is the wetting phase saturation in the reservoir.

Here $S_w = (1 - S_{Hg})$, and take the logarithm of the fractal geometry formula, we get:

$$\lg(1 - S_{Hg}) = (D - 3) \lg P_c + (3 - D) \lg P_{\min} \quad (3)$$

The logarithmic coordinates of $(1 - S_{Hg})$ and P_c are drawn, and the fractal dimension D is obtained by the slope of the line.

In order to analyze the heterogeneity of pores with different pore sizes, the relationship curves of S_{Hg}/P_c and S_{Hg} were drawn to segment the log-double graph of the fractal model based on the high-pressure mercury injection method [8-10], as shown in Figure 3. Because of the low initial injection pressure, the mercury saturation grows extremely slowly in the first stage. The curve rapidly lowers as the mercury saturation slowly increases, showing that larger holes are currently being invaded by mercury. As the injection pressure increases, more pores with decreasing sizes become occupied by mercury, increasing the saturation of mercury. and the curve gradually rises to a low point in the graph that represents the pressure P_{ce} . After a while, when mercury enters smaller pores, the capillary pressure P_c rises along with the mercury saturation S_{Hg} . However, the rate at which P_c increases is rather quick, resulting in a peak-forming reduction in S_{Hg}/P_c . The pressure P_{ca} is represented by this peak. The double logarithmic curve graph of $S_{Hg} \sim P_c$, $(1 - S_{Hg}) \sim P_c$ is separated into three parts of larger pores, medium pores, and smaller pores using the test points near P_{ce} and P_{ca} as turning points. Each segment is then fitted and analyzed independently to determine the corresponding fractal dimensions.

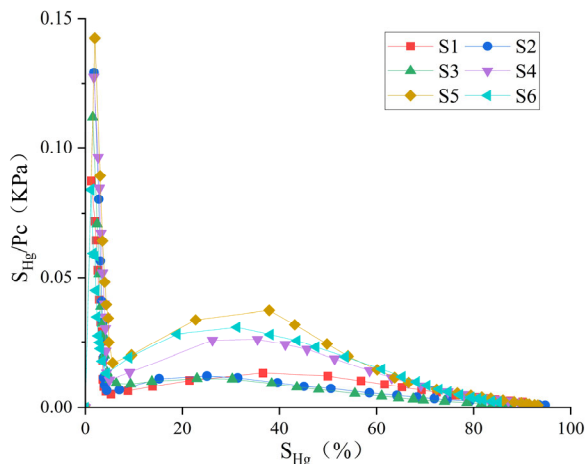


Figure 3. Relation curve of S_{Hg}/P_c and S_{Hg} in sandstone sample.

The fractal characteristics of S1 sandstone sample are shown in Figure 4. S_{Hg} and P_c values are directly derived from mercury intrusion porosimetry data, and Figure 4 illustrates the creation of a double logarithmic graph of $(1 - S_{Hg})$ and P_c by merging the fractal model. The fractal curve shows a distinct three-segment structure with strong linearity, suggesting that the fractal properties vary with pore size. The linear curve's various segments' fractal dimensions are shown by the variables D_{b1} , D_{b2} , and D_{b3} .

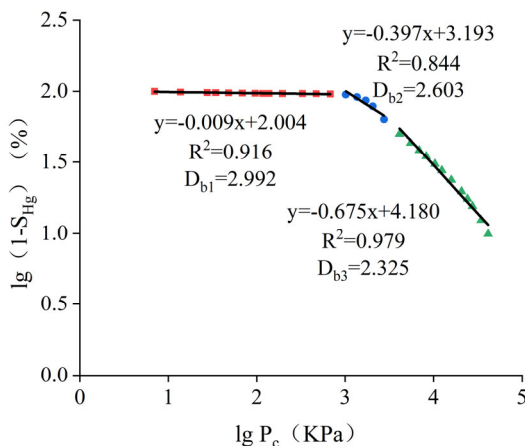


Figure 4. S1 sample fractal model curve characteristics

4. Permeability Prediction Model Based on Fractal Theory

In order to establish a permeability prediction model based on fractal theory, the following assumptions are made: (1) The sandstone pores are assumed to be composed of a series of tortuous tube bundles. (2) Assume that the aperture distribution is continuously differentiable. (3) It is assumed that the length of the capillary bundle also presents fractal characteristics. (4) Set in a porous medium, some particles overlap each other, while some particles do not overlap. According to the above assumptions, the calculation formula of the required parameters is obtained. According to the fractal theory, there is a power exponential relationship between the number of capillary bundles and the capillary diameter:

$$N(\geq \lambda) = \left(\frac{\lambda_{\max}}{\lambda}\right)^{D_f} \tag{4}$$

Where λ is the pore diameter; $N(\geq \lambda)$ is the number of pores whose pore diameter is greater than or equal to λ . λ_{max} is the maximum pore diameter. D_f is the fractal dimension.

The fractal dimension D_t of tortuosity and the feature length L_0 are obtained from the following formula:

$$D_t = 1 + \frac{\ln \tau}{\ln(L_0/\bar{\lambda})} \tag{5}$$

$$L_0 = \left[\frac{1-\varphi}{\varphi} \times \frac{\pi D_f \lambda_{max}^2}{4(3-D_f)} \right]^{1/2} \tag{6}$$

Where, $\bar{\lambda}$ is the average pore diameter; τ is the average tortuosity; φ is the porosity.

According to Darcy's Law, permeability calculation prediction models can be obtained in the following ways:

$$K = \frac{\pi}{32(D_t+3)} \frac{L_0^{1-D_t}}{A} \frac{D_f}{3+D_t-D_f} \lambda_{max}^{3+D_t} \tag{7}$$

The tortuosity fractal dimension can be used to measure the curvature of the path through which the fluid flows in a tight sandstone core. The expression of the fractal dimension D_t and the mean tortuosity $\bar{\tau}$ is as follows :

$$D_t = 1 + \frac{\ln \tau}{\ln L_0/(2r_{av})} \tag{8}$$

$$\bar{\tau} = \frac{1}{2} \left[1 + \frac{1}{2} \sqrt{1-\varphi} + \sqrt{1-\varphi} \cdot \frac{\sqrt{\left(\frac{1}{\sqrt{1-\varphi}}-1\right)^2 + \frac{1}{4}}}{1-\sqrt{1-\varphi}} \right] \tag{9}$$

Where, $\bar{\tau}$ is the tortuosity of the curved streamline; r_{av} is the average capillary radius; L_0 is the characteristic length of the capillary.

According to the above calculation formula, the capillary characteristic length L_0 , the average tortuosity $\bar{\tau}$ and the fractal dimension D_t of tortuosity were calculated respectively, and the permeability prediction K_y was calculated combined with the fractal dimension D_f , the maximum pore throat radius r_{max} and the average pore throat radius r_{av} obtained from the mercury injection experiment. The results are shown in Table 1.

Table 1. Parameter data required for permeability prediction model based on fractal theory

Sample	D_f	r_{max}	r_{av}	L_0	$\bar{\tau}$	D_t	K_y
S1	2.461	0.722	0.228	2.948	8.231	2.129	0.004
S2	2.430	1.094	0.250	4.562	5.519	1.773	0.026
S3	2.592	1.086	0.238	3.960	8.231	1.995	0.017
S4	2.564	1.569	0.461	5.880	8.381	2.148	0.022
S5	2.562	2.244	0.621	8.428	5.984	1.934	0.060
S6	2.645	2.229	0.631	7.656	7.789	2.139	0.037

In the study of establishing a permeability prediction model based on fractal theory, it is found that the permeability predicted by the model has a large deviation from the measured permeability, and the predicted permeability value is expected to be small. However, the predicted permeability value predicted based on fractal theory has a strong correlation with the measured permeability value, and the fitting coefficient reaches 0.932, as shown in Figure 5.

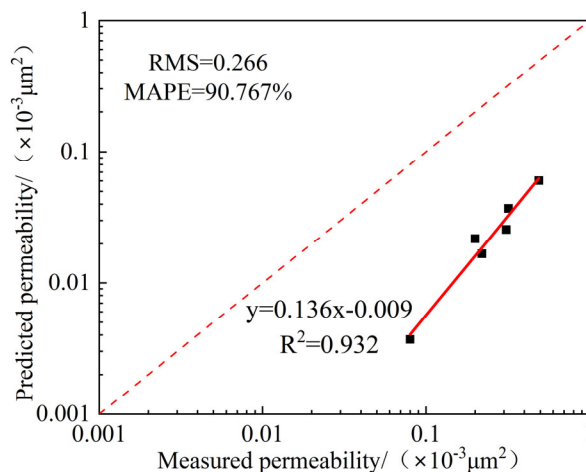


Figure 5. Analysis of permeability prediction model based on fractal theory

The prediction effect of the permeability prediction model based on fractal theory was evaluated. The root mean square value (RMS) was 0.266, and the mean absolute percentage error (MAPE) was 90.767%. The prediction effect was not satisfactory, which may be due to the assumption that the model based on fractal theory was based on ideal conditions.

The permeability prediction value was revised based on the fitting relation. The relation between the revised permeability prediction value and the original prediction value was as follows:

$$K_x = 7.353K_y + 0.066 \tag{10}$$

The prediction error of the revised permeability prediction value was calculated, in which the RMS value was 0.031, the RMS value was reduced by 88.496%, the MAPE value was 11.929%, and the average absolute percentage error was reduced by 86.857%. The prediction accuracy is obviously improved, which is more suitable for sandstone permeability prediction in the Linxing area.

5. Conclusion

The pore fractal of sandstone has the characteristics of three-stage, and the fractal dimension can be used to analyze the heterogeneity of sandstone pore structure effectively. Tortuosity and fractal dimension of tortuosity are the key to predicting infiltration by applying high pressure mercury injection and fractal theory. After modifying the permeability prediction model, the prediction accuracy is obviously improved, which is more suitable for sandstone permeability prediction in Linxing area.

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