

A Summary of the Application of Mathematics in the Design of Quadrotor UAV

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Abstract

With the rapid development of UAV technology, mathematical methods play a vital role in the design of quadrotor UAV. This paper systematically expounds the application of mathematical tools in mathematical modeling, control algorithm and optimization method of quadrotor UAV. Aiming at the nonlinear characteristics of UAV, such as the torque model has no driving force characteristics and strong coupling, Euler-Lagrange equation is used to establish a six-degree-of-freedom model of UAV, and coordinate transformation theory is used to accurately describe the motion of UAV. In the control algorithm, the classical PID control, adaptive control, modern model predictive control (MPC), sliding mode control and other technologies are integrated, among which nonlinear MPC(NMPC) combined with sequential quadratic programming (SQP) algorithm significantly improves the trajectory tracking accuracy. Intelligent control field, neural network and fuzzy control also increase the possibility of independent judgment for complex environment. By optimizing the route design and voyage duration, this paper effectively improves the efficiency and stability of flight. The research shows that the comprehensive application of mathematical methods lays a good mathematical foundation for the control accuracy and flight endurance of quadrotor UAV.

Keywords

Quadrotor UAV, mathematical modeling, control algorithm, model predictive control.

1. Introduction

With the rapid development of UAV technology, mathematical methods play an increasingly important role in the design of quadrotor UAV. The quadrotor UAV is a typical underactuated, strongly coupled and nonlinear system (see Figure 1), and its design process involves complex mathematical modeling, control algorithm and optimization method. Euler-Lagrange equation used when creating the most basic dynamic model; Various mathematical methods used in flight control include proportional integral differential (PID) control; Adaptive control based on Lyapunov stability theory: Unpredictable control method in the face of uncertain factors and sliding mode control for nonlinear properties. In recent years, with the rapid development of computer, more and more advanced mathematical methods have been applied to the design of quadrotor UAV. Model predictive control (MPC) has gained extensive attention because it combines optimization theory with control theory. Especially for the nonlinear model predictive control (NMPC), a new way of solving the nonlinear dynamic characteristics of quadrotor UAV is provided by constructing a nonlinear programming problem and combining with numerical optimization methods such as sequential quadratic programming (SQD). In addition, related mathematical theories in artificial intelligence and machine learning, such as neural network and fuzzy logic, have also been applied to the autonomous control system and decision-making system of quadrotor UAV. The comprehensive application of these mathematical means can not only improve the control accuracy and system performance of quadrotor UAV, but also provide theoretical basis for the application of quadrotor UAV in various industries[1].

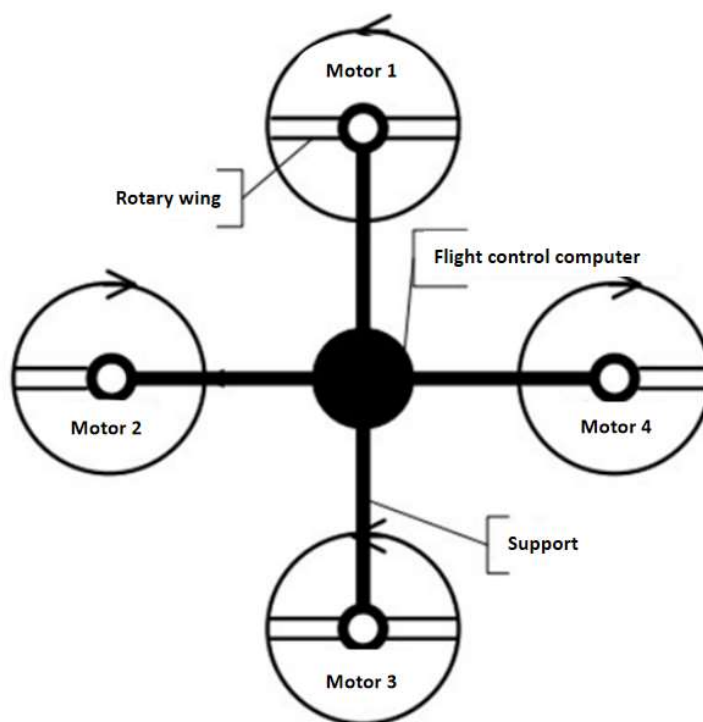


Figure 1. Structure of quadrotor UAV

2. Mathematical Modeling of Quadrotor UAV

2.1. Coordinate System and Transformation

In the mathematical modeling of quadrotor UAV, the establishment of coordinate system and coordinate transformation are the most basic mathematical tools. The "X" quadrotor UAV platform has four independent propeller motors, so it is a three-dimensional working object with six degrees of freedom (three translations and three rotations). In order to accurately describe the motion of UAV, two coordinate systems are established: inertial coordinate system (called ground coordinate system) and airframe coordinate system. Among them, the inertial coordinate system can often be set on the ground, and the right-handed rectangular coordinate system is selected, with the Z axis pointing to the sky and the X axis horizontal to the Y axis. Put the coordinate system of quadrotor UAV body at the center of mass, where the X axis is along the front direction of the body, the Y axis is along the right direction of the body, and the Z axis is perpendicular to the ground direction, following the right-hand rule. The four propellers are located at the intersection of the X axis and the Y axis with an included angle of 45 degrees in the same human coordinate system, forming an "X"-shaped distribution. Yaw angle ψ (around Z axis), pitch angle θ (around Y axis) and roll angle φ (around X axis) are the transformations between two coordinate systems. With the help of the combination of the above three angles of rotation matrix around the axis, the mutual transformation between the body coordinate system and the inertial coordinate system can be realized, which is the mathematical basis for the subsequent establishment of dynamic equations and the design of control laws (see Figure 2). According to the practical engineering application, this paper sets the quadrotor UAV as a completely rigid and symmetrical model, and the center is located at the geometric center. This can simplify the mathematical model, but still reflect most of the model characteristics.

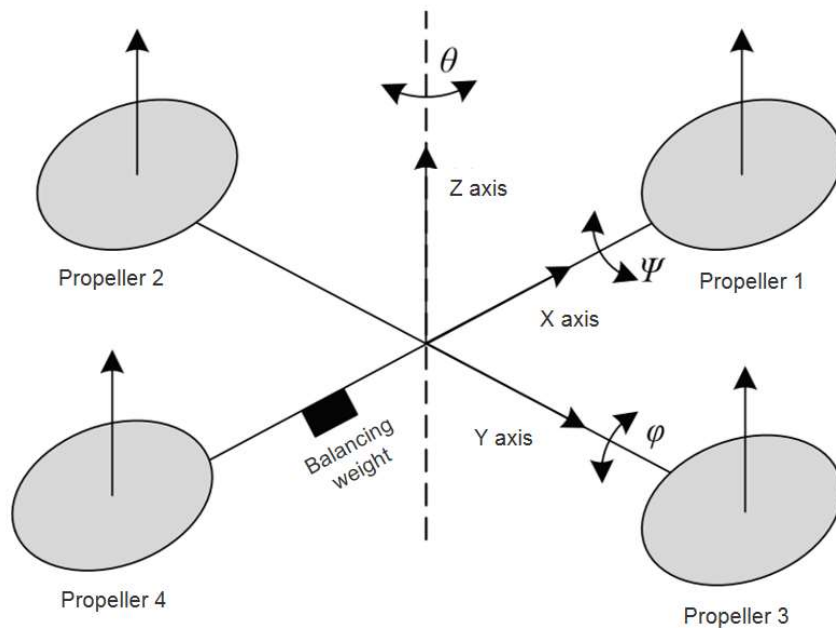


Figure 2. Coordinate system of quadrotor UAV body

2.2. Aerodynamic Model

Propeller thrust and torque are the core elements of flight control of quadrotor UAV, and their mathematical relationship profoundly affects the flight performance and stability of UAV. As a key variable, the propeller speed (Ω) not only determines the lift, but also controls the rotating attitude of the UAV. In the generation of total thrust, the sum of the squares of the rotational speeds of the four propellers directly determines the vertical lift of the UAV, that is, the total thrust F_z is proportional to $\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2$. This nonlinear characteristic shows that the small change of rotating speed will significantly affect the lift, making the UAV flexible to adjust its height. For example, when all propellers accelerate synchronously, the drone will climb rapidly; And synchronous deceleration will lead to decline.

In the process of torque control, the asymmetric installation of propellers will produce different torsional forces, and the different rotational speeds of the front and rear propellers will produce lifting forces, which will cause the UAV to retreat or advance; When the rotational speeds of the left and right propellers are different, the sliding force will be generated, which will make the UAV perform tilting flight. When the left and right propellers cross, turning force will be generated, which is convenient to control the forward direction of the UAV. All the above torques play a key role in maintaining the balanced posture and accurate displacement of the UAV [2].

The efficiency of propeller can be calculated according to mathematical model and algorithm, and the coupling relationship between propeller speed, thrust and torque can be effectively analyzed, which can provide sufficient theoretical basis for designing stable flight control.

2.3. Dynamic Modeling

In the process of dynamic modeling of quadrotor UAV, Euler-Lagrange equation is the core tool to build accurate model, which can comprehensively and meticulously establish a six-degree-of-freedom nonlinear model of UAV (covering the position and posture in three-dimensional space), and deeply describe the various forces suffered by UAV during flight motion and the complex relationship between these forces and motion.

In the real flight scene, due to the different shapes of UAVs, if all the small quantities are considered, the established UAV model is very complicated and inconvenient to manage. The rigid body hypothesis can simplify the model and retain the necessary dynamic characteristics.

Euler-Lagrange equation studies problems from the perspective of energy. The equation analyzes the dynamic characteristics of UAV in detail, including its kinetic energy and the relationship between kinetic energy and the motion rate and rotation speed of UAV in three-dimensional space, and points out the energy States existing in the operation of UAV. Similarly, the equation analyzes that the static energy of UAV is mainly determined by the height of UAV in space, which shows the energy of UAV running on the earth. This precise analysis method is used to avoid the constraint problem. The previous modeling methods need to add a lot of redundant constraints when solving the binding force, which greatly increases the difficulty of solving the equation.

The dynamic model of UAV based on Euler-Lagrange equation can accurately present the nonlinear and strong coupling characteristics of quadrotor UAV in flight, which provides a solid theoretical support for the subsequent design of efficient and stable control algorithm [3].

3. Mathematical Algorithm in the Control Algorithm of Quadrotor UAV.

3.1. Classical Control Algorithm

The theoretical basis of the control system of quadrotor UAV is the traditional control algorithm: PID control, adaptive control, robust control and other control technologies. Pid control is the most widely used basic algorithm. For the control of four-axis UAV, PID controllers are used in position control loop and attitude control loop respectively, and the attitude and position of UAV can be accurately controlled by setting three different gain values. The adaptive control algorithm can adjust the control parameters independently according to the change of the working state and external conditions of the system, thus improving the anti-interference and adaptability of the system [4]. The core idea of adaptive control technology is to ensure that the system is always in the optimal control state through online parameter estimation and controller parameter adjustment. Robust control strategy is to consider the parameter variation of the controlled object and the robustness of its external influencing factors in the design, and to optimize the transfer function of the controller by using mathematical methods such as H^∞ control theory and μ synthesis technology to ensure the reliability and effectiveness of the control performance of the system under uncertain conditions. Although these classical control algorithms can't meet the strong nonlinear characteristics of quadrotor UAV, their simple mathematical structure and strong engineering practical significance still make them have great practical value.

3.2. Intelligent Control Algorithm

The development of control technology of quadrotor UAV based on advanced intelligent control algorithm is mainly carried out from the aspects of neural network control, fuzzy control and reinforcement learning control. These algorithms are based on artificial intelligence, machine learning theory, combined with data-driven thinking to learn and optimize control strategies to achieve a new idea to solve automatic control in complex environment. Neural network control method uses the nonlinear mapping and self-learning ability of artificial neural network to generate the black box model of UAV system, and realizes the learning of input-output strategy in the whole UAV system control process. Fuzzy control method establishes UAV fuzzy rule base and reasoning mechanism through fuzzy logic theory, and mathematicizes expert knowledge to realize uncertain and nonlinear problems in control process. Reinforcement learning control uses the method of agent-environment interaction and reward function strategy gradient to optimize the control strategy of UAV control system and realize the ability of autonomous control and self-adaptation of UAV [6]. These algorithms include neural network theory, fuzzy logic, probability theory, optimization theory and other mathematical means, and advanced methods of UAV control are given by using different mathematical tools.

Although the existing research has not involved the details of implementation, this algorithm shows great development potential in enhancing the autonomy, adaptability and robustness of UAV, and has become an important direction of UAV control technology development in the future[5].

4. Application of Mathematical Optimization Algorithm in Quadrotor UAV.

4.1. Trajectory Planning Optimization

For quadrotor UAV system, trajectory planning optimization is one of the core technologies of control, which provides the optimal trajectory for quadrotor UAV under the conditions of dynamic limitation and safety [7]. A complete dynamic model of quadrotor UAV is designed, and two common routes are designed as test trajectories to verify the feasibility of nonlinear model predictive control method. First, the reference trajectory of spiral descent, the mathematical expression of which is:

$$\begin{cases} x = \sin(t) \\ y = \cos(t) \\ z = t \end{cases}$$

This path constructs a curve of three-dimensional spiral descent (see Figure 3), which can verify the tracking ability of UAV to complex three-dimensional trajectory.

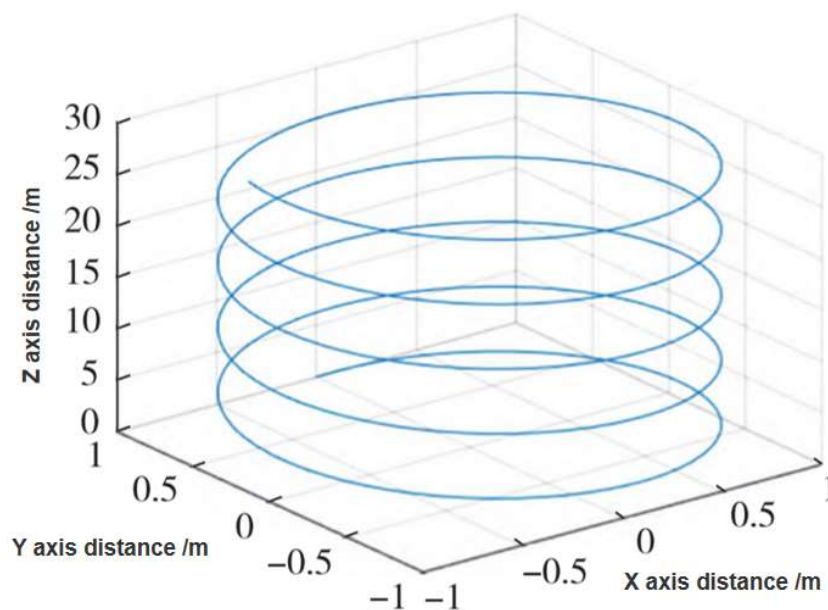


Figure 3. Reference track 1

The second is the horizontal "8" reference trajectory, and its mathematical expression is:

$$\begin{cases} x = \sin(t) \\ y = \sin(t) \\ z = \text{ones}(\text{size}(t)) \end{cases}$$

This trajectory constitutes a plane 8-shaped track (see Figure 4), which can be used to verify the performance of UAV track position accuracy in complex plane tracks.

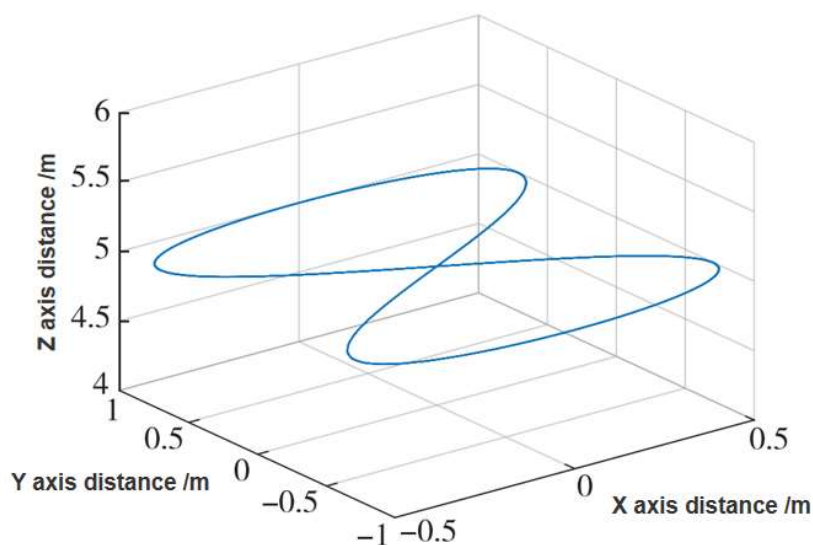


Figure 4. Reference track 2

When planning the path, we fully consider the dynamic constraints of quadrotor UAV, such as control input constraint range $U_{\min} < U < U_{\max}$, state constraints, etc., to ensure that the obtained path can be executed in reality. In order to verify the rationality of the control scheme in this paper, we first determine the initial position coordinates $[x, y, z, \dot{x}, \dot{y}, \dot{z}] = [0, 0, 0, 0, 0, 0]$, the simulation running time $t=30s$, and the time slot of each step is $T=0.1s$, which shows that the NMPC control mode can approach the target orbit within 3s, and the maximum trajectory tracking error value is less than 0.05m. This method can provide a set of mathematical modeling methods for various complex scenes and effectively solve the problem of self-driving flight of quadrotor UAV.

4.2. Energy Optimization

In the quadrotor UAV system, there is a kind of mathematical problem of energy optimization, which aims to reduce energy consumption as much as possible to meet the requirements of flight performance and achieve longer flight time. In order to minimize flight energy consumption, firstly, the dynamic equation of quadrotor UAV is defined: $F_z = \alpha \sum_{i=1}^4 \omega_i^2$, where α is the thrust coefficient and ω_i is the rotational speed of the i th propeller. The model describes the relationship between the total thrust generated by four propellers of quadrotor UAV and the sum of the squares of each propeller speed, which provides a mathematical basis for energy optimization. In the aspect of motor power distribution, the rotational speed of four propellers $[\omega_1^2, \omega_2^2, \omega_3^2, \omega_4^2]$ is taken as the control input U , and the power output of each motor is reasonably distributed through the optimization algorithm to ensure the minimum energy consumption while meeting the flight requirements. By setting the control input constraint $U_{\min} < U < U_{\max}$, the power distribution of the motor is ensured in a safe range, and the system instability caused by excessive power is avoided. According to the simulation experiment, the designed controller makes the rotation speed of propeller $\leq 10r/s$, which meets the control accuracy and achieves a good effect of reducing energy consumption, and provides a mathematical basis for long-term flight of quadrotor unmanned aerial vehicle.

4.3. Controller Parameter Optimization

For the control system design of quadrotor UAV, the core of mathematical problem is to optimize the parameters of the controller, and its ultimate goal is to get better control effect through optimized parameters. At this time, the nonlinear model predictive controller is used, and through the optimization of various objectives, the state tracking error is small and the constraint conditions of controlling input energy are met as the main objectives. Optimization setting expression of objective function:

$$\begin{aligned}
 j &= \sum_{i=1}^N \|x_p(k+i-1) - x_r(k+i-1)\|_Q^2 \\
 j &= \sum_{j=1}^N \|U(k+i-1) - x_r(k+i1)\|_R^2
 \end{aligned} \tag{1}$$

Where J is the objective function, $x_p(k)$ is the predicted state quantity, $x_r(k)$ is the expected state quantity, U is the system input quantity, Q and R are the state weight matrix and the input weight matrix, respectively. $Q \in A^6 \times 6$, $R \in A^4 \times 4$. The first half of the objective function is to accurately track by minimizing the difference between the predicted value and the expected value weighted square, and the second half is to save energy by minimizing the input weighted square. In order to prevent the quadrotor motor from getting out of control due to high voltage, the control input is strictly limited in the processing of constraint conditions, which are expressed as: $U_{\min} < U < U_{\max}$, where U_{\min} is the minimum input matrix and U_{\max} is the maximum input matrix. This constraint design ensures that the operation input is within the allowable range and keeps the system running stably. In order to get the strategy, sequential quadratic programming (SQP) is used to solve NLP problem online. This method can effectively solve the nonlinear optimization problem with constraints, and the optimal solution of quadratic programming molecules in the subprogram is obtained through iteration to gradually approach the optimal solution of the original problem. SQP method has the advantages of fast convergence and good numerical stability, and is especially suitable for online optimization of real-time time control systems. Through the design of controller parameters based on mathematical optimization, the trajectory tracking and attitude control of quadrotor UAV are completed with high precision, which provides a mathematical theoretical basis for UAV to fly smoothly in complex environment.

5. Conclusion

In this paper, the key role of mathematical methods in the design of quadrotor UAV is analyzed in detail. Aiming at the nonlinear problem of unbalanced force and high coupling of UAV, a six-degree-of-freedom dynamic model is established by Euler-Lagrange equation, and an accurate correspondence between inertia and dynamics is established by coordinate transformation. In the control design, conventional PID, adaptive controller, model predictive control and sliding mode controller are introduced, especially nonlinear MPC(NMPC) combined with sequential quadratic programming (SQP) effectively enhances the tracking accuracy (mean square error $< 0.05m$). In addition, in intelligent supervision, it is an intelligent control method to deal with this problem based on neural network and fuzzy logic. Through this method, a new way of self-decision in more complicated problems can be solved. Through this scheme, path planning and energy efficiency scheme are combined to improve flight power and steady state. The research proves that the deep integration of mathematical tools can effectively break through the nonlinear constraints of UAV and provide theoretical support for the precise control and long-term flight of industrial UAV.

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