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MLGM: A Novel Gravity Model for Node Importance Evaluation based on the Integrated Characteristics of Nodes

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Abstract

Identifying key nodes in complex networks is a fundamental and challenging problem in network science, and it has garnered widespread interest over the past few years. The existing centrality methods in assessing node importance still are inadequate in comprehensively utilizing node features, and struggle to achieve optimal performance across different network types. In response, this paper proposes a gravity centrality based on node comprehensive features and node weight. The model incorporates the local information and positional attributes of a node into its quality, and considers using the maximum eigenvalue as the node's weight to reflect its global influence in the network, which effectively addresses the inherent heterogeneity of nodes. To validate the accuracy and effectiveness of this model in identifying key nodes, the paper compares the proposed gravity centrality with traditional centrality methods across six real-world network datasets from multiple evaluation perspectives. The experimental results confirm the strong precision and effectiveness of the proposed method in recognizing pivotal nodes.

Keywords

Identification of Key Nodes, Gravity Centrality, Multiple Attributes, Node‘s Weight.

1. Introduction

The research of complex networks has gained significant attention within network science, facilitating the modeling and analysis for multiple types of real-world networks. Common networks in real society include power networks[1], biological networks[2], social networks[3], transportation networks[4]and computer networks[5]among others. In complex networks, key nodes contribute significantly to shaping both the structural integrity and functionality of the network. The more influential a node is, the more critical its role within the network. Accurately identifying important nodes is essential for preventing traffic congestion and accidents, optimizing ecosystem management and conservation, understanding the dynamics of information diffusion or disease transmission[6]reducing the risk of power systems failure, and enhancing the security of computer networks.

In the analysis of complex networks, identifying and evaluating key nodes has always been an important research direction. Most classical centrality methods for identifying important nodes rely on two primary aspects: neighborhood topology and path-based. On one hand, typical neighborhood-based methods comprise Degree Centrality (DC)[7], K-Shell Decomposition Centrality (KS)[8] Improved K-Shell Decomposition Centrality (IKS)[9], Mixed Degree Decomposition (MDD)[10]and Extended Neighborhood Coreness (Cnc+)[11]. However, each of these methods has its own advantages and limitations. DC measures node importance by counting the number of its direct neighbors, but it fails to account for the network’s overall configuration. The KS is widely used in identifying core and peripheral nodes, but the node ranking it generates can be rather coarse, and there is no clear distinction between nodes with

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identical KS value. To resolve this issue, Wang et al. presented the IKS, which introduces node information entropy to better differentiate nodes within a single shell layer. IKS is a hybrid metric that strikes a balance between local and global measures. Zeng et al.[10] proposed the MDD, which involves the residual and discarded neighbors in the K-Shell decomposition, and incorporated a parameter λ with adjustable settings. However, MDD presents a challenge in that the selection of the optimal λ is highly dependent on the network topology. The Cnc+ method, proposed by Bae et al.[11] , considers both direct and indirect neighbors (i.e., neighbors of neighbors) of nodes, capturing nodes with potential for information propagation in the network. These approaches emphasize the importance of the shortest paths between vertices in the context of information propagation. Nevertheless, applying them to large networks can lead to a considerable escalation in computational demands.

To find solutions to these limitations of the aforementioned classical methods, this paper uses the gravity-based centrality method to identify key nodes. Gravity centrality was first proposed by Ma et al.[12]based on Newton's law of universal gravitation. They pointed out that power of a node is contingent on its immediate neighbors as well as on non-neighboring nodes, with interactions typically diminishing as the shortest path length becomes larger. Therefore, according to the gravity centrality they proposed, the k-shell value of each node represents its mass, with the distance specified by the shortest path length. Compared to traditional centrality methods, the gravity centrality approaches can incorporate both path information and the neighborhood topology structure of nodes as part of their mass, allowing for a more comprehensive assessment of node importance. Expanding upon the gravity centrality discussed above, Li et al.[13] raised the Local Gravity Model (LGM), which revealed an empirical linear interplay regarding the optimal truncation radius and the network's average distance. By incorporating a truncation radius, the LGM effectively reduced computational complexity while maintaining accuracy. Unlike Ma's model, LGM uses a node's degree as its mass. Additionally, Yang et al.[14] introduced the KS based on Gravity Centrality (KSGC), which give thought to the node's position, significantly enhancing the identification of important nodes. Zhu et al.[15] developed the H-index-based gravity centrality (HVGC) based on h-index, incorporating both path information and node positional information, thus allowing for more effective identification of nodes with small k-shell values.

On one hand, the improvement focuses on enhancing the node quality aspect. Li et al.[16]submitted an Effective Gravity Model (EGM), using the node's spreading ability as the factor in calculating its mass. Lv et al.[17] used the PageRank centrality value as node's mass. Li et al.[18] measured local information through the local clustering coefficient and node's degree value. In the Multi-Characteristics Gravity Model (MCGM), Li et al.[19] used node's degree value, eigenvector centrality value and k-shell value as indicators for calculating node's mass. Guo et al.[20] utilized the degree and k-shell values to obtain the local and positional characteristics of nodes. On the flip side, the improvement focuses on the inter-node distance. Yang et al.[21] advanced the Gravity Centrality method based on an Adaptive truncation radius and Omni-channel paths (AOGC), where the inter-node loose distance is worked out from Omni-channel paths. Ren et al.[22] proposed the Feasible Path-based Local Gravity Model (FPLGM), where the distance is calculated as the reciprocal of the number of viable paths connecting the nodes. Jiang et al.[23] replaced the shortest distance with a probability-incentivized effective range to uncover implicit dynamic information within the network.

However, most existing gravity centrality methods have the following two limitations: (1) The majority of gravity models still have limitations in fully incorporating the node characteristics and information from different perspectives of nodes to evaluate their importance. (2) many of the previous gravity centrality methods fail to account for the heterogeneity of node. To overcome these limitations, this paper introduces a novel gravity centrality method-MLGM: Gravity Model based on Multiple attributes and Largest eigenvalue as node weight-which aims

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to more accurately identify key nodes within a network. The main contributions of this method are as follows: (1) This paper uses both node degree and clustering coefficient to capture the local structural features of nodes. The degree of a node intuitively reflects the number of its neighboring nodes, serving as the most basic indicator for measuring local information of the node. The clustering coefficient, which indicates the degree of local clustering around a node, also impacts the node's ability to propagate information in network dynamics. (2) Since the IKS and MDD methods can differentiate the importance of nodes with the same KS value, effectively addressing the issue where KS fails to distinguish the importance of nodes within the same shell layer, this paper utilizes the improved information entropy from the IIKS method and the mixed degree from the MDD method to assess the positional information of nodes. (3) When a node has greater influence in the network, its interactions with other nodes tend to be stronger. However, many previous gravity centrality methods have overlooked the inherent differences between nodes. To address node heterogeneity, this paper assigns a weight to each node based on its largest eigenvalue, while reflecting the node's global information within the network.

After establishing the MLGM, simulations are conducted on six real-world network data sets from various perspectives, including propagation dynamics and network robustness analysis. Classical centrality methods (DC, BC, KS, IKS, MDD) and previous gravity centrality methods (G, LGM, ILGM, MCGM) focus on different network structures and node attributes. Therefore, by comparing MLGM with these centrality methods can highlights the distinctive characteristics of MLGM from multiple angles and comprehensively validates its effectiveness. The final experimental results verify the high accuracy and effectiveness of MLGM in identifying key nodes. Thus, this paper provides a more comprehensive and precise analytical tool for identifying and evaluating important nodes.

The following sections outline the structure of this work: Section 2 presents comprehensive sketch of the centrality methods for discerning crucial nodes previously mentioned, including classical centrality measures and gravity centrality methods. Section 3 explains the specific calculation process of MLGM in detail. Section 4 presents simulation experiments from various perspectives to verify the high accuracy and effectiveness of the MLGM in identifying important nodes. Finally, Section 5 summarizes the main content of the paper and discusses some existing limitations and prospective research topics.

2. Preliminaries

In this chapter, we describe the fundamental symbols and formulas, classical centrality methods used for evaluating node importance, and gravity centrality proposed in previous research. Firstly, $G(V,E)$ illustrates an elementary network, where $V=\{v_1,v_2,...,v_n\}$ describes the ensemble of nodes within the network, $|V|=n$ signifies the total count of nodes; $E=\{(v_i,v_j)\,|\,v_i,v_j\in V\}$ represents the assemblage of edges, $|E|=m$ describes the total count of edges. The adjacency matrix of the network is symbolized by $A=(a_{ij})_{n\times n}$ where $a_{ij}=1$ indicates that an edge connects node i with node j, $a_{ij}=0$ represents that the two nodes are not linked.

2.1. The Classic Measures for Identifying Key Nodes

2.1.1. Degree Centrality

Degree Centrality (DC)[24]functions as a basic measure of node centrality, which assesses the node’s significance based on how many edges it has with other nodes. It can be referred to as:

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$$DC(i) = \frac{\sum_{j=1}^{n} a_{ij}}{n-1} = \frac{k_i}{n-1} \tag{1}$$

where k_i demonstrates the degree value of the node i.

2.1.2. Betweenness Centrality

Betweenness Centrality (BC)[25] measures the frequency with which a node functions as an intermediary in every shortest path of the network. The prominence of a node in the structure increases with the quantity of routes that traverse it. The betweenness centrality can be formulated as:

$$BC(i) = \frac{\sum_{j \neq i \neq w \in V} \frac{\varpi_{jw}(i)}{\varpi_{jw}}}{(n-1)(n-2)/2} \tag{2}$$

where ϖ_{jw} signifies the quantity of shortest paths from node j to w, and $\varpi_{jw}(i)$ captures the total number of shortest paths across node i. $(n-1)(n-2)/2$ works for normalizing the betweenness centrality.

2.1.3. K-Shell Decomposition

K-Shell Decomposition (KS) [8] measures the positional importance of nodes in a network by iteratively removing nodes based on their degree. The principle of this method is as follows: first, identify all nodes with a degree of 1 in the network and remove these nodes along with their connected edges. Then, update the degree values of the remaining nodes. Continue repeating the above steps until no node with a degree of 1 remains in the network. At this point, all removed nodes constitute the first layer, i.e., the 1-shell. By analogy, this process continues until all nodes are assigned a corresponding k-shell value. A higher value indicates that the node is located in a more central "shell" within the network structure. However, since this method cannot distinguish the importance among nodes within the same shell, Wang et al. [9] later proposed the IKS method based on it.

2.1.4. Improved K-Shell Decomposition

The Improved K-Shell Decomposition Method (IKS)[9] computes the information entropy and factors in the nodes' position. Assuming the criticality of the node i is I_i, the information entropy is equivalent to:

$$I_i = \frac{k_i}{\sum_{j=1}^{n} k_j} \tag{3}$$

$$e_i = -\sum_{j \in \Gamma(i)} I_j \ln I_j \tag{4}$$

where $\Gamma(i)$ shows the collection of neighbors associated with node i.

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2.1.5. Mixed Degree Decomposition Method

The Mixed Degree Decomposition Method (MDD)[10] performs K-Shell decomposition in accordance with the mixed degree of the residual and the eliminated neighbor nodes throughout the network, effectively improving the discrimination of the KS method. Let $k_i^{(r)}$ and $k_i^{(e)}$ represent the residual and removed degree values for node i, respectively. Therefore, the mixed degree can be written as:

$$k_i^{(m)} = k_i^{(r)} + \lambda * k_i^{(e)} \tag{5}$$

which λ takes values between 0 and 1. At $\lambda=0$, the method aligns with the KS method, and at $\lambda=1$, it becomes equivalent to the DC. The parameter value used here is 0.7.

2.2. The Measures based on Gravity Model

2.2.1. Gravity Model

Ma et al.[12] firstly put forward a gravity centrality method, which takes the k-shell value of the node i to represents its mass and the distance between nodes is interpreted with regard to the shortest path length. The formula is:

$$G(i) = \sum_{j \in \psi_i} \frac{ks(i)ks(j)}{d_{ij}^2} \tag{6}$$

where ψ_i signifies the set of nodes adjacent to node i, with distances less than or equal to r. Here, r can be set to 3.

2.2.2. Local Gravity Model

Building on the Ma et al.'s survey, Li et al.[13] introduced an improved local gravity model (LGM), which effectively uses neighborhood topology and path information to find pivotal spreaders. Thus, it is expressed as:

$$LGM(i) = \sum_{d_{ij} \le R, j \ne i} \frac{k_i k_j}{d_{ij}^2} \tag{7}$$

where the R refers to the truncation radius, here setting $R=2$ can achieve optimal results.

2.2.3. Improved Local Gravity Model

Wu et al[26] proposed an Improved Local Gravity Model (ILGM) based on the LGM. By integrating the paths and topological features of neighboring nodes, the accuracy of the results was significantly enhanced.

$$ILGM(w) = \sum_{h \in \tau_w} LL(h) \tag{8}$$

where, τ_w is the first-order neighborhood of node, and

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$$LL(h) = \sum_{d_{ht} \le R, t \ne h} \frac{H(h)H(t)}{d^2(h,t)} \tag{9}$$

$$H(h) = \sum_{i \in \tau_h} LGM(i) \tag{10}$$

2.2.4. Multi-Characteristics Gravity Model

Li et al.[19] offered a multi-characteristics gravity model (MCGM) that aggregate multiple node attributes to determine the node's mass, and calculates the shortest path length as the distance between nodes. The resulting equation is:

$$MCGM(i) = \sum_{d(i,j) \le R, j \ne i} \frac{\left(\frac{k_i}{k_{\max}} + \frac{\alpha ks_i}{ks_{\max}} + \frac{ec_i}{ec_{\max}}\right)\left(\frac{k_j}{k_{\max}} + \frac{\alpha ks_j}{ks_{\max}} + \frac{ec_j}{ec_{\max}}\right)}{d_{ij}^2} \tag{11}$$

$$\alpha = \frac{\max\{\frac{k_{mid}}{k_{\max}}, \frac{ec_{mid}}{ec_{\max}}\}}{\frac{ks_{mid}}{ks_{\max}}} \tag{12}$$

Where $R = 2$, the maximum of the degree, k-shell value, and eigenvector centrality value are represented as $k_{\max}$, $ks_{\max}$ and $ec_{\max}$, respectively. Value α is introduced to reduce the effect exerted by the k-shell value, where k_{mid}, ks_{mid} and ec_{mid} represent their median values.

3. Literature References

As mentioned earlier, existing centrality methods based on gravity models have two limitations. On the one hand, traditional gravity centrality methods mostly focus on the nodes' local or global features when assessing their importance, but they lack the ability to comprehensively integrate node features and information from different perspectives. On the other hand, the node heterogeneity is vital in identifying key parts, yet most existing gravity-based centrality methods fail to account for this issue.

To deal with these two issues, the section proposes the gravity centrality that incorporates multiple node features and weights to identify influential spreaders. The model posits that a node's importance is not only determined by its local information but is also influenced by its positional information and global information. Specifically, the degree and clustering coefficient are used as indicators reflecting local structural features, while the node's improved information entropy and mixed degree are employed to represent the node's positional information in the network. Then, the local information and positional information of the node are integrated to calculate the node's mass. Additionally, this paper considers using the largest eigenvalue as the node's weight to capture its global information within the network.

3.1. Calculation Framework of MLGM

The MLGM model consists of these key components: the mass of the node, the weight assigned to each node, and the shortest path and truncation radius between nodes. Figure 1 illustrates the algorithmic framework of the proposed MLGM, which is described in detail below.

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Algorithm 1 : Framework of the proposed MLGM method.

Input: Graph G(V,E)
Output: Gravity centrality MLGM for each node
begin

Phase 1 Calculating node mass
Calculating node mass based on multiple property dimensions, including node's local information and location information in the network
Phase 2 Calculating node weight
The maximum eogenvector value is introduced as node weight and reflect global information in the network
Phase 3 Calculating interaction distance between nodes
Use the shortest path length to measure interaction distance between nodes, and to control the effective interaction distance between nodes within a reasonable range, calculate truncation radius
Phase 4 Calculating node interaction force and gravity centrality
Step 1 Computing node interaction that one node imposed on another based on their local information and location information
Step 2 Calculating the force imposed by one node on another
Step 3 Getting the gravity centrality of each node

return Gravity centrality MLGM

Figure 1. The calculation framework of the MLGM

3.2. Improved Node Information Entropy

Information entropy takes into account the positional information of nodes and comprehensively considers the propagation effect of a node on its first-order neighbors. The higher the node information entropy value, the easier it is for the node to propagate its influence to neighboring nodes, and thus the greater the node's influence. This method can address the shortcomings of the traditional K-shell method. It can be used to rank the importance of nodes with the same K-shell value and accurately identify nodes with greater influence. However, the current node information entropy only considers the first-order neighbors of a node, which is insufficient for some real-world networks, as second-order neighbors often play a non-negligible role in propagation dynamics. Based on this, this section proposes an improved information entropy (IIKS), which extends the calculation of node information entropy by simultaneously considering both the first-order and second-order neighbor nodes of a node, thereby reflecting a broader propagation potential. Additionally, because second-order neighbors may include nodes connected by multiple first-order neighbors, redundant information should be avoided in the calculation. To this end, its specific expression is as follows:

$$e(i) = -\left[\sum_{j \in \Gamma_1(i)} I_j \ln I_j + \sum_{k \in \Gamma_2(i)} \omega(i,k) I_k \ln I_k \right] \tag{13}$$

In the equation, j represents the first-order neighbor nodes of node , and k represents the second-order neighbor nodes of node i. $\Gamma_1(i)$ is the first-order neighborhood, and $\omega(i,k)$ is the second-order neighborhood of node . is the decay weight formula, which is designed to reflect the fact that in real-world networks, when influence propagates to second-order neighbor nodes, its intensity decreases with increasing distance. Therefore, a decay weight formula should be applied to second-order neighbor nodes based on their distance. Its expression is as follows:

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$$\tilde{\omega}(i,k)=\tilde{\beta}\frac{path(i,k)}{\max_{k\in\Gamma_2(i)} path(i,k)} \tag{14}$$

In the equation, $\tilde{\beta}$ represents the decay factor, with a value of 0.5. $path(i,k)$ represents the number of common first-order neighbor nodes that connect node i to its second-order neighbor node. When it equals 1, it indicates that node i and its second-order neighbor node are connected through a unique first-order neighbor node; otherwise, it indicates that there are multiple first-order neighbor nodes between them.

3.3. Calculating of Node Mass

Step 1 Calculating local information of nodes.

This section employs node degree and clustering coefficient as indicators to measure the local information of nodes. The degree of a node reflects the number of its neighboring nodes. Nodes with high degree value are referred to as "key nodes" or "hub nodes", and removing these nodes could lead to the collapse of the network. Therefore, to some extent, the degree can reflect the vulnerability or stability of the network. The clustering coefficient[27] indicates the strength of local clustering around a node. A higher clustering coefficient suggests that a node's neighbors are more densely connected, forming a tighter-knit cluster. Conversely, a low clustering coefficient suggests fewer connections between neighboring nodes, resulting in a more loosely connected structure. Additionally, the clustering coefficient influences a node's ability to spread information in dynamic processes[28]. Considering both node degree and clustering coefficient to evaluate the influence of nodes can enhance the accuracy of node ranking[29]. Thus, we utilize the node degree and clustering coefficient as indicators to measure the local information of nodes.

Step 2 Calculating positional information of nodes.

The strength of interaction between a node and others is closely linked to the its position within the network. Nodes located closer to the network's core generally have a greater potential to influence the system[30]. Therefore, the positional information of a node plays an irreplaceable role in identifying important nodes. The KS is the most common technique for determining node's position. However, its ranking results are too rough, as it fails to precisely distinguish nodes within the same shell that may have varying levels of influence. To address this issue, we employ IIKS method. Additionally, the MDD method assesses node importance by considering multiple node attributes (including degree, residual degree, and depleted degree). Compared to KS method, MDD provides a more comprehensive reflection of node's position and influence by incorporating multiple dimensions of measurement. Therefore, in this paper, we combine the improved information entropy from the IIKS method and the mixed degree from the MDD method to represent a node's positional information.

Step 3 Calculating of node mass.

The local information and positional information of the nodes are integrated to determine the node's quality. The specific calculation formula is as follows:

$$mass(i)=k_i+Cc(i)+e(i)+k_i^{(m)} \tag{15}$$

where k_i represents the node degree, $Cc(i)$ represents the clustering coefficient of the node, $e(i)$ denotes the improved information entropy of the node, and $k_i^{(m)}$ represents the mixed degree. Since these indicators are not on the same scale, they need to be normalized. Thus, the equation can be rewritten as:

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$$mass(i) = \frac{k_i}{k_{\max}} + \frac{Cc(i)}{Cc_{\max}} + \frac{e(i)}{e_{\max}} + \frac{k_i^{(m)}}{k^{(m)}{}_{\max}} \tag{16}$$

By combining these four indicators to assess node quality, this approach offers several advantages compared to other centrality methods: (1) Enhanced precision in node importance evaluation. By incorporating multiple attributes from both local and positional information, which offers an integrated evaluation of a node's overall impact, significantly improving accuracy of importance evaluation. (2) Finer differentiation among nodes. By utilizing improved information entropy and mixed degree-two indicators that effectively capture a node's positional information-this method enables finer differentiation of nodes within the same shell. These advantages allow for a more nuanced analysis of each node's surrounding environment, promoting a more accurate measurement of node influence.

3.4. Calculating of Node‘s Weight

The impact of each node in a real-world network is not identical. For example, in social networks, some individuals with strong social skills may have a large number of friends, while others with fewer social interactions may have smaller social network. This difference means that certain spreaders are more powerful in information propagation or contagion spreading. The larger the effect of a node, the stronger its interaction with other nodes. However, some previous studies on gravity centrality methods did not taken into account the inherent differences between nodes. Therefore, to address the issue of node heterogeneity, this section assigns a weight to each node based on its maximum eigenvector value, reflecting the global information of the node within the network.

$$AX = \lambda X, ec(i) = X_i \tag{17}$$

in the formula, λ represents the maximum eigenvalue and X denotes the normalized eigenvector, $ec(i)$ indicating the value of the corresponding node i in the vector X_i.

3.5. Calculating Distance between Nodes and Truncation Radius

The inter-node distance is a critical determinant in calculating gravity centrality. In this section, by evaluating the shortest path length, we determine the inter-node distance. Meanwhile, a truncation radius[13] is employed to limit the interaction range between nodes. There are two reasons for using a truncation radius: First, in practical propagation processes, the interaction strength between distant nodes is often estimated inaccurately, as long-distance interactions tend to accumulate errors, which reduces the overall precision of the calculations. Second, calculating the distance in a large-scale network would be computationally expensive. Therefore, this paper adaptively determines a truncation radius R with reference to the interaction distances between nodes, and computes the gravity centrality within the limited interaction range. This section sets $d_{ij} \le R, j \ne i$ as the calculation condition, while set assume that the influence range generated by each node does not exceed half of the network diameter.

3.6. Calculating Gravity Centrality

Step 1 Calculate interactions between nodes.

According to Newton's law of universal gravity, the force F(i, j) exerted by one node on another can be defined as:

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$$F(i,j)=\frac{mass(i)\times mass(j)}{d_{ij}^{2}} \tag{18}$$

Where $mass(i)$ and $mass(j)$ are the masses of nodes i and j respectively, and d_{ij} is the shortest path length between the two nodes.

Step 2 Calculate the gravity centrality value of nodes.

To calculate the influence of node$i$, the total force exerted by the node on all its neighboring nodes within a given range must be summed, while assigning a weight to each node based on its maximum eigenvector value. Therefore, the formula for MLGM is as follows:

$$MLGM(i)=ec(i)\times\sum_{d_{ij}\le R,\,j\ne i}F(i,j) \tag{19}$$

where R represents the truncation radius of the network.

Therefore, the MLGM model can be written as follows:

$$MLGM(i)=ec(i)\times\sum_{d_{ij}\le R,\,j\ne i}\frac{\left(\frac{k_i}{k_{\max}}+\frac{Cc(i)}{Cc_{\max}}+\frac{e(i)}{e_{\max}}+\frac{k_i^{(m)}}{k^{(m)}{}_{\max}}\right)\left(\frac{k_j}{k_{\max}}+\frac{Cc(j)}{Cc_{\max}}+\frac{e(j)}{e_{\max}}+\frac{k_j^{(m)}}{k^{(m)}{}_{\max}}\right)}{d_{ij}^{2}} \tag{20}$$

4. Numerical Experiments

In complex network analysis, identifying key nodes allows for the proactive reinforcement of critical parts of the network, thereby enhancing the system's overall resilience to disturbance. There are various methods employed to identify important nodes, including those based on network diffusion dynamics models and those based on network robustness analysis. In diverse evaluation models, the significance of nodes depends on varied evaluation systems. The network diffusion dynamics models (such as the SIR model) hold the view that node's diffusion capacity is responsible for determining node's importance. If the spread of a disease or information within the system is larger and faster, it indicates that the nodes carrying the virus or information are more influential. In contrast, the network robustness analysis focuses on the extent of network collapse caused by nodes that are intentionally attacked or fail. The more severe the network's collapse, the greater the impact of these nodes. Therefore, this paper will assess the ability of the MLGM method to identify key nodes from both of these perspectives, utilizing multiple evaluation criteria.

4.1. Experimental Framework

This section analyzes the performance of the proposed MLGM for pinpointing critical spreaders using six real-world network data sets. The paper compares MLGM with several classic centrality methods (DC, BC, KS, IKS, MDD), as well as gravity-based centrality models proposed in previous research (G, LGM, ILGM, MCGM), to verify the precision of MLGM in recognizing significant nodes within complex networks. Section 4.2 provides detailed description of basic topological structure about six actual networks. Section 4.3 outlines the techniques implemented in the simulation experiments, including the SIR model, the Kendall correlation coefficient, robustness metrics, and monotonicity indicator. In the 4.4 segment, the Kendall correlation coefficient serves to compare the node rankings generated by various centrality methods with those derived from SIR model, providing a measure of the accuracy of node

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ranking in each method. Section 4.5 utilizes robustness metrics to analyze the network collapse resulting from intentional node removal, testing the effectiveness of each centrality method in detecting critical spreaders. Section 4.6 evaluates the efficiency of MLGM methods in detecting vital nodes using evaluation criteria such as monotonicity indicator and centrality score. Finally, the paper concludes by summarizing all experimental results, demonstrating the superiority and exactness of MLGM in the detection of crucial nodes.

4.2. Network Overview

To analyze the reliability of MLGM, we conduct comparative experiments on six real-world network data sets. These data sets include Jazz, USAir, Email, Messages, Router, and PB, which come from different fields and application scenarios. The Jazz data set[31] represents a collaboration network based on the cooperation between jazz musicians. The USAir data set models the transportation network of USAir, where the edges indicate direct flights between airports. The Email data set[32] describes the network of email exchanges between workers at Rovira i Virgili University in Spain. The Messages data set[33]refers to a network created by an online community at the University of California, Irvine, spanning from April to October 2004. The Router data set[34] represents a network of routers within a technical infrastructure. The PB (Political Blogs) [35]data set is a hyperlink network of political blogs in the United States.

Table 1 describes the fundamental topological characteristics about the six networks. Here, n and m represent the count of nodes and edges; $<k>$ refers to the average degree; $<d>$ represents the average path length (or called average distance); Cc shows the network's clustering coefficient; the network diameter is denoted as d . The heterogeneity index of the degree distribution is represented as H , calculated using the formula: $H=<k^2>/<k>^2$. The propagation threshold is represented as β_c.

Table 1. Basic topological characteristics of six complex network data sets

Network	n	m	$<k>$	$<d>$	Cc	H	β_c
Jazz	198	2742	27.70	2.24	0.63	1.40	0.027
USAir	332	2126	12.81	2.74	0.75	3.46	0.023
Email	1133	5451	9.62	3.61	0.25	1.94	0.057
Messages	1266	6451	10.19	3.31	0.068	0.57	0.038
Router	2113	6632	6.28	4.61	0.25	0.61	0.049
PB	1222	16714	27.36	2.74	0.36	2.97	0.013

4.3. Relevant Models and Methods

4.3.1. SIR Model

The SIR model[36] is a widely used propagation dynamic model in complex networks. Due to its reasonable and effective simulation capability, the SIR model serves to assess the performance of various methods in identifying important nodes. The SIR model assigns nodes to several states: susceptible, infected, and recovered. The specific diffusion process unfolds as follows: disease or information spreads through contact of vulnerable and infected individuals at a certain infection rate. Infected individuals then transition to either recovered or deceased situation at a certain chance. This diffusion procedure persists until state transitions cease, at which point the system reaches a steady state. In this section, we will apply the classic SIR model from EoN library in Python to mimic the diffusion course. Furthermore, the infection rate is closely related to the network's diffusion threshold, denoted as β_c . The equation for computing the diffusion threshold below is:

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$$\beta_c = \frac{<k>}{<k^2> - <k>} \tag{21}$$

here, $<k^2>$ corresponds to the second moment of the degree distribution[37].

4.3.2. Kendall Correlation Coefficient

The definition of the Kendall correlation coefficient[38] is as follows: We first assume there are two sequences $X = (x_1, x_2, ..., x_N)$ and $Y = (y_1, y_2, ..., y_N)$ with the same number of elements. The definition of the coefficient is given below: Assume there are two pairs of tuples, if $(x_i - y_i)(x_j - y_j) > 0$, then the rankings of these two tuples are consistent. Conversely, if $(x_i - y_i)(x_j - y_j) < 0$, then the rankings of these two tuples are inconsistent. Based on this, the formula can be presented below:

$$\tau = \frac{2(N_c - N_d)}{N(N-1)} \tag{22}$$

which N_c signifies the count of pairs of observations that are consistently ranked across the two data sets. When a pair of data points in each sequences maintains the same relative order, the count is incremented by 1. N_d represents the number of pairs of observations that are inconsistently ranked. When a pair of data points in each sequence has a different relative order, the count is increased by 1. The correlation coefficient τ lies within the range [-1,1], with τ=1 representing perfect positive correlation, τ=-1 indicating perfect negative correlation, and τ=0 denoting no correlation.

4.3.3. Robustness Metric

Robustness metric[39] is a widely used validation methods for assessing the significance of nodes within a network. It evaluates the accuracy of various techniques for identifying critical nodes by simulating deliberately attacking on the network's certain nodes. This involves removing certain nodes according to their centrality measure values, and then assessing the extent of network collapse following each removal. In this section, the collapse is quantified using the robustness metric value, which refers to the relative magnitude of the largest connected component in the residual network after nodes removal. If its scale decreases more rapidly after nodes' removal, it indicates a more severe collapse, suggesting that the attacked nodes are more critical and the approach proves to be precise and powerful in pinpointing critical nodes. Conversely, if the collapse occurs more slowly, it implies that the current method is less effective at pinpointing key nodes. The robustness measure can be indicated as:

$$MR = \frac{1}{n} \sum_{Q=1}^{n} S(q) \tag{23}$$

which $1/n$ serves as a normalization term for comparing the robustness metric of networks of varying scales. The symbol $q = Q/n$ denotes the proportion of nodes that have been removed, while $S(q)$ represents the relative scale of the maximum connected component within the remaining network after node removal. The robustness metric measures the region between the $S(q)$ curve and coordinate axes. If the area of this region is smaller, it reflects superior capability of the current centrality method in detecting pivotal nodes.

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4.3.4. Monotonicity Index

The monotonicity index[40] is typically employed for contrasting the discrimination competence of various centrality methods in analyzing node's significance, with a greater emphasis on maintaining global monotonicity. The definition of the monotonicity index is as follows:

$$M(L)=[1-\frac{1}{n(n-1)}\sum_{l\in L}nl(nl-1)]^2 \tag{24}$$

which L represents the ranked score list obtained from different centrality methods, and nl denotes the quantity of nodes with matching importance. The value of $M(L)$ ranges from (0,1). When $M(L)=0$, it indicates that a particular ranking list has many nodes on equal footing, signifying that present approach offers the weakest differentiation ability. In contrast, when $M(L)=1$, it indicates that all elements in a ranking list have unique ranks, demonstrating that it excels in node identification.

4.4. Ranking Accuracy

We investigate the effectiveness and precision of MLGM for node importance ranking in this part. The closer the ranking results are to the actual propagation capabilities of the nodes, the more effective the method is at identifying critical nodes. Firstly, from the perspective of network propagation dynamics, the SIR model serves to replicate the actual transmission patterns in the network nodes, which produces a standard ranking. The standard ranking is derived by treating each node as an initial infected node and performing a contagion simulation. The nodes ranking is determined by the proportion of infected and recovered individuals once the network reaches a steady state. To assure the reliability of the outcomes, we carry out 1,000 separate experiments with each node serving as the initial infected node, then the simulation results are averaged. In addition, the Kendall correlation coefficient is deployed to measure congruence between the node rankings produced by each method and the standard ranking. Since the Kendall correlation coefficient lies within the range [−1,1], the coefficient closer to 1 indicates that node importance ranking is more accurate. Therefore, the next step is to calculate the Kendall correlation coefficient between the rankings sourced from each centrality approach and the SIR standard ranking under varying infection rates.

4.4.1. When Infection Rate is Constant ($\beta = \beta_c$), the Kendall Correlation Coefficients Corresponding to Each Centrality Method

Table 2. Kendall correlation coefficients

Network	DC	BC	KS	IKS	MDD	G	LGM	ILGM	MCGM	MLGM
Jazz	0.826	0.472	0.804	0.899	0.846	0.823	0.871	0.868	0.916	*0.917*
USAir	0.782	0.579	0.809	0.887	0.790	0.816	0.874	0.846	0.883	*0.885*
Email	0.801	0.645	0.817	0.889	0.810	0.840	0.876	0.813	0.907	*0.910*
Messages	0.831	0.721	0.838	0.901	0.833	0.886	0.885	0.860	0.902	0.891
Router	0.706	0.513	0.741	0.845	0.713	0.826	0.856	0.791	0.877	*0.889*
PB	0.865	0.692	0.875	0.894	0.870	0.831	0.887	0.886	0.901	*0.912*

As can be clearly observed in Table 2, the gravity model demonstrates a clear advantage in Kendall's tau coefficient compared to traditional centrality methods. In the Jazz, USAir, Email, Router, and PB networks, the node ranking results of MLGM show a high correlation with those of the SIR model, indicating that MLGM achieves the highest accuracy in identifying important

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nodes. In the Messages network, even though MLGM is not the best-performing method, its Kendall's tau coefficient remains relatively high. Among the classical centrality methods, the IKS method performs most outstandingly. The reason lies in IKS's ability to more precisely identify the core structures and hierarchical relationships within the network by calculating improved information entropy. Among the gravity centrality methods, MLGM exhibits the highest Kendall's tau coefficient, followed by MCGM, suggesting that their node rankings are the closest to the standard ranking. In summary, across all networks, MLGM demonstrates the highest precision in node ranking.

4.4.2. When Infection Rate Varies (β/β_c is within the Range [0.5, 1.5]), the Changes in Kendall Correlation Coefficients Corresponding to Each Method

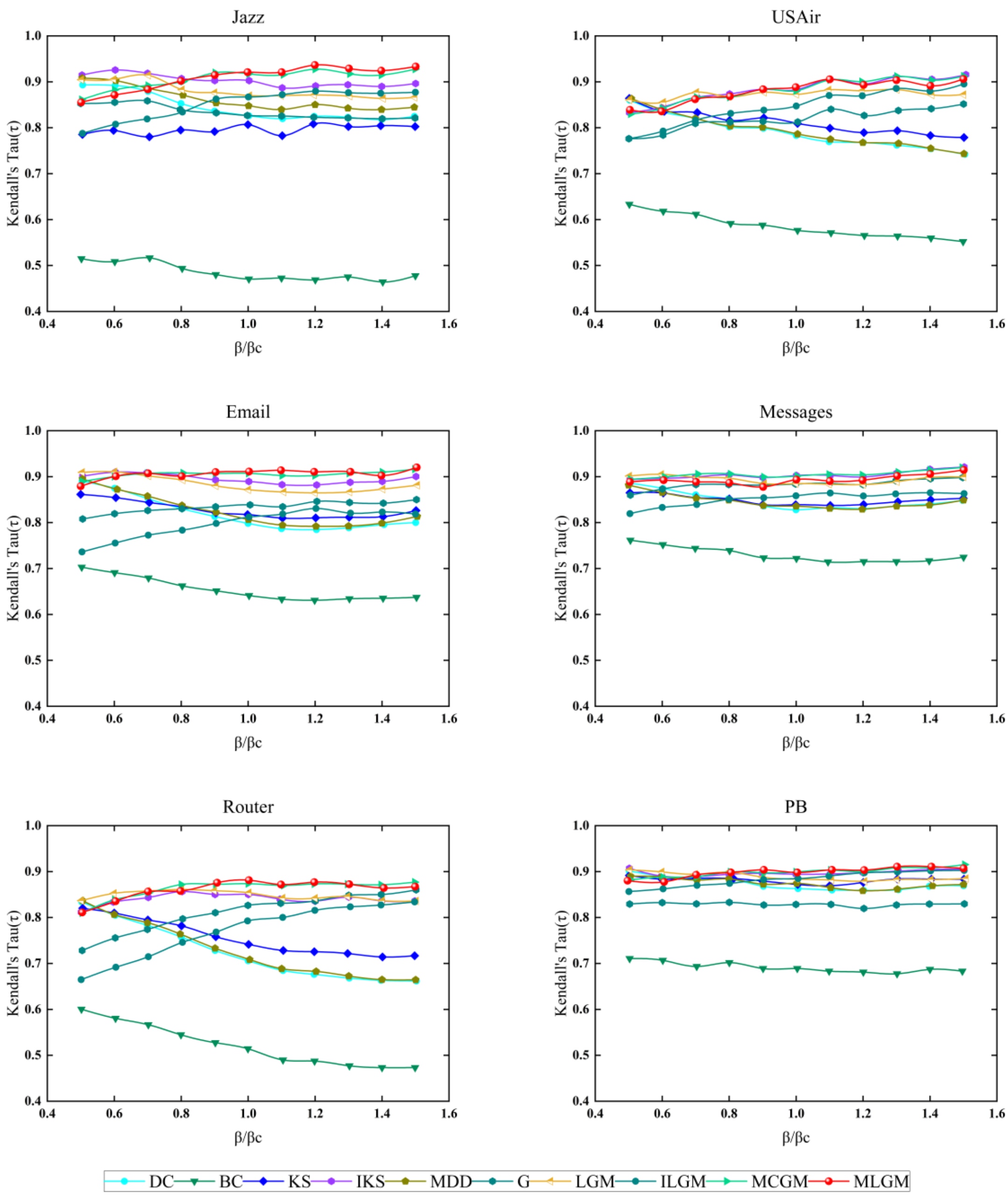

Figure 2. Variation of Kendall correlation coefficients on six real networks

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To enhance the realism of the results, this section conducts experiments by adjusting β in the SIR simulation process, and it is closely linked to the propagation threshold β_c. The infection rate directly impacts the accuracy of the experiments, when β is excessively large, disease may spread too rapidly throughout the entire network, creating obstacles in exactly gauging the nodes' impact. Conversely, when β is too low, the spread will be restricted to a smaller area, which also hampers an effective evaluation of their roles[41]. Given the variations in network size and structure, this section sets the infection rate β to be close to the propagation threshold β_c for the experiments. Specifically, β is defined as $h \times \beta_c$, where *h* ranges from 0.5 to 1.5. Thus, β/β_c falls within the interval [0.5, 1.5], with an increment of 0.1. During the propagation simulations, we will use the ratio β/β_c to cover a range of infection rate variations. After conducting 1000 simulations for each experiment, the statistical average will be computed. If the Kendall correlation coefficient for a particular centrality method is remarkably high, it points out that the node ranking obtained through the centrality closely resembles the standard ranking derived from the SIR model, demonstrating the method's high accuracy in ranking important nodes. The experimental results for the six real networks will be presented in Figure 2.

As can be seen from Figure 2, MLGM significantly outperforms other methods in identifying important nodes in almost all networks, indicating its strong advantage in key node detection. Conversely, BC performs the worst in distinguishing node influence, consistently ranking below all other methods. In the Jazz network, when $\beta/\beta_c > 0.8$, MLGM achieves the highest Kendall's tau coefficient. In the USAir, Email, Router, and PB networks, the node ranking obtained by MLGM is almost the closest to the standard ranking, followed by IKS and MCGM. In the Messages network, although MCGM achieves the highest accuracy in identifying key nodes, MLGM also delivers strong performance, ranking second only to MCGM. In summary, across these networks with diverse structures and properties, the ranking list generated by MLGM is highly correlated with the standard node ranking derived from SIR model simulations of actual propagation capability. This demonstrates that MLGM can better distinguish nodes with varying influence levels, enabling effective and accurate ranking of node importance.

4.5. Robustness Metric

We assess the efficacy of various methods in determining key nodes via observing robustness metric after eliminating designated constituents. The smaller robustness metric corresponding to a particular method (i.e., the smaller the closed region in Figure 3), the faster the network collapses after specific nodes removal, which implies the attacked nodes are more vital and the method's ability to identify important nodes is stronger. The robustness measure results obtained in this section are shown in Figure 3, the X-axis shows the fraction of designated nodes removed, and the Y axis denotes the relative size of the maximum connected subgraph remaining in the network after the node removal. The space demarcated by each curve along with the X and Y axes represents robustness measure corresponding to the current method.

As shown in Figure 3, in most networks, the robustness curve of the MLGM method is almost at the bottom, with the corresponding maximum connected region shrinking rapidly as nodes are eliminated. This indicates that removing just a few nodes can cause a considerable network collapse, suggesting that MLGM is highly effective at pinpointing key nodes. A comprehensive analysis of the robustness metrics across all networks revels that MLGM results in the greatest network collapse, with the smallest area between its curve and the coordinate axes. This demonstrates that the key nodes selected by MLGM have a more substantial impact on the network, making MLGM the most effective method for identifying important nodes.

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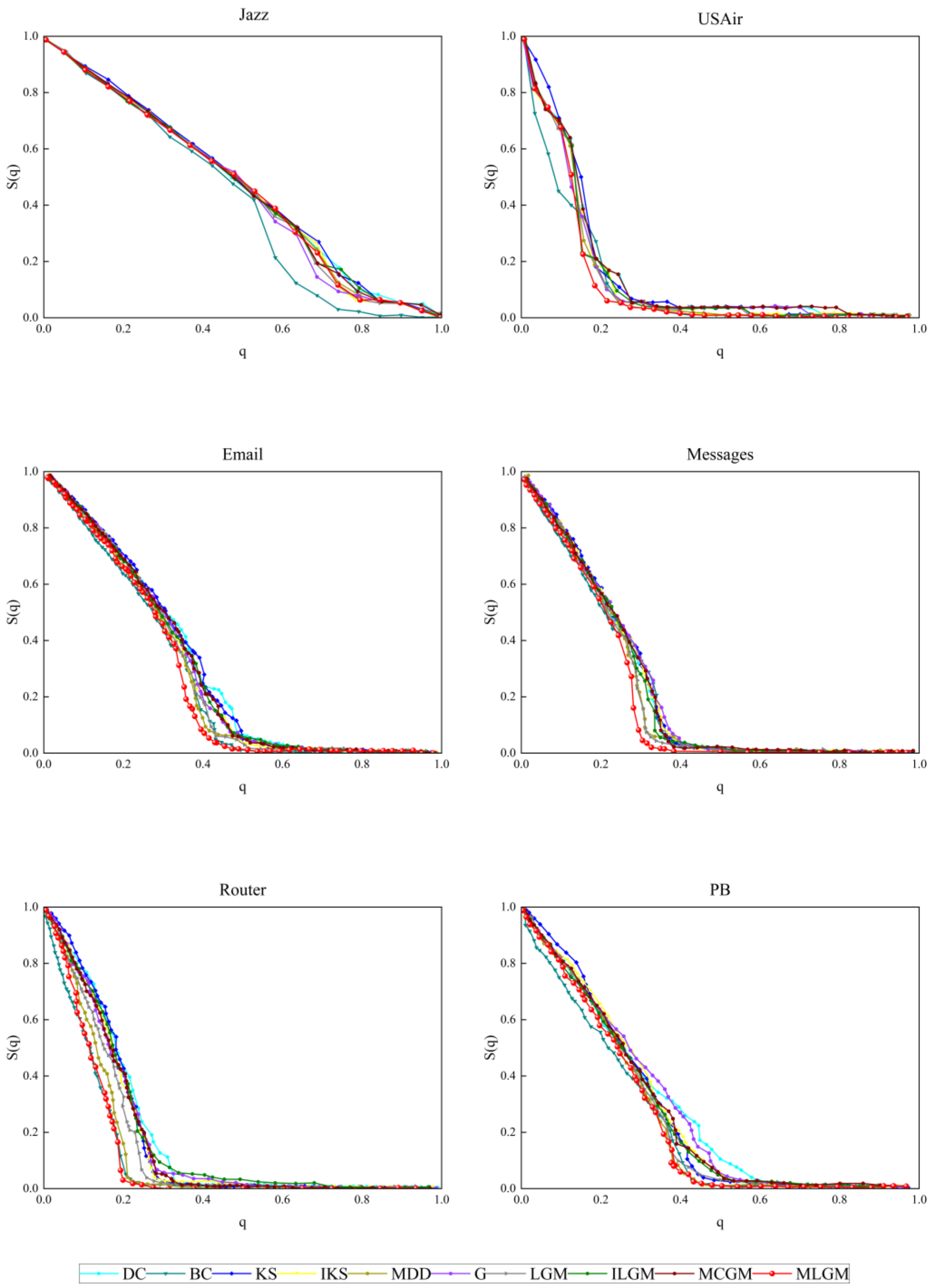

Figure 3. Robustness analysis graph

4.6. Distinguishing Ability

This section uses a monotonicity index to contrast the effectiveness of various centrality approaches to differentiate node significance. The larger the monotonicity index, the more capable the approach is at distinguishing nodes of varying magnitude within the network. The monotonicity index values for various centrality methods on 8 real networks are presented in Table 3.

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Table 3. Monotonicity index values

Network	DC	BC	KS	IKS	MDD	G	LGM	ILGM	MCGM	MLGM
Jazz	0.9659	0.9886	0.7944	0.9995	0.9927	0.9988	0.9992	0.9991	0.9994	*0.9996*
USAir	0.8586	0.6970	0.8114	0.9942	0.8879	0.9931	0.9950	0.9952	0.9954	0.9951
Email	0.8874	0.9400	0.8088	0.9990	0.9234	0.9998	0.9997	0.9998	0.9998	*0.9999*
Messages	0.8604	0.9230	0.7759	0.9984	0.8893	0.9998	0.9997	0.9997	0.9998	*0.9998*
Router	0.7206	0.6692	0.6489	0.9922	0.7569	0.9990	0.9984	0.9990	0.9993	*0.9993*
PB	0.9324	0.9480	0.9060	0.9992	0.9448	0.9992	0.9992	0.9992	0.9992	*0.9993*

5. Conclusion

Classical centrality methods and most previous gravity-based approaches still face limitations in comprehensively utilizing various node characteristics and information when assessing node importance. In response to the limitation, we present a new gravity model based on multi-attribute and weight of nodes (MLGM). This method incorporates the degree and clustering coefficient of a node to reflect its local structural features, calculates the node's improved information entropy and mixed degree to indicate its positional information, and incorporates the node's maximum eigenvector value as the node's weight to reflect its global information. Ultimately, to verify the capability of MLGM to detect vital nodes, simulations are conducted upon six real-world network data sets, evaluated from multiple testing perspectives and evaluation criteria. The findings of the experimental demonstrate the effectiveness and exactness of MLGM at recognizing critical nodes.

The simulation in this paper adopts two distinct testing perspectives: infectious dynamics and network robustness analysis. By comparing MLGM with other centrality methods, the following conclusions are drawn: (1) Accuracy of node ranking. The accuracy of node ranking is evaluated by calculating the Kendall correlation coefficient for each method. The results show that under a certain infection rate, MLGM achieves the highest Kendall correlation coefficient across all networks. (2) Precision of identifying important nodes. The correctness is assessed by deliberately attacking specific nodes in the network and using robustness metrics to determine the proficiency of these approaches in detecting crucial nodes. The experimental results demonstrate that in all networks, the largest connected subgraph corresponding to MLGM decreases the fastest as the proportion of deleted nodes increases, indicating that MLGM excels at identifying important nodes. (3) Node discrimination ability. Firstly, the monotonicity index for each method is calculated, and the results revel that the node ranking produced by MLGM exhibits high monotonicity, which indicates that MLGM has the superior discrimination ability. Additionally, centrality scores for various methods are computed, and the results conclude that MLGM has a strong ability to differentiate nodes. From the overall experimental results, it can be concluded that the MLGM proposed in this paper performs excellently in node ranking accuracy, node identification ability, and discrimination capability.

Building upon previous research, a gravity centrality is introduced for pinpointing crucial nodes by integrating multi-dimensional information of nodes. However, despite the significant results achieved by the MLGM in measuring node importance, future research will consider more perspectives of node information to more accurately assess critical component within network. Moreover, the assessment of vital nodes in higher-order networks remains a pressing issue to be addressed. Therefore, in subsequent studies, we will place more emphasis on research related to higher-order networks, further enhancing the applicability of gravity-based centrality methods to comprehensively tackle the diverse challenges in complex networks.

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