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Study on the Water Quality Evaluation of the Yiluo River based on Improved Fuzzy Comprehensive Evaluation

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Abstract

Accurate river water-quality assessment is essential for watershed management, yet fixed-threshold and static weighting schemes may not adequately represent the fuzziness and gradual transitions of water-quality states. This study proposes a composite-weighted improved fuzzy comprehensive evaluation framework and applies it to the state-controlled Qilipu section of the Yiluo River using high-frequency monitoring data from 2022–2024. Five indicators are weighted via CRITIC and entropy methods and integrated into composite weights; semi-trapezoidal membership functions based on GB 3838-2002 are used to construct fuzzy relation matrices and derive comprehensive evaluation vectors. The results classify the study section as Class II in all three years under the maximum membership principle, indicating generally good water quality. Interannual weight and membership patterns further identify NH_3–N and TP as key constraints on improvement toward Class I, suggesting nutrient control as a management priority. Overall, the proposed approach provides an objective and interpretable workflow for translating monitoring data into water-quality grades and limiting-factor diagnostics.

Keywords

Water-quality Assessment, CRITIC Weight, Entropy Weight, Fuzzy Comprehensive Evaluation.

1. Introduction

The Yiluo River is a major first-order tributary in the middle reaches of the Yellow River, supporting multiple functions such as regional agricultural irrigation, urban development, and ecological security. With accelerated urbanization and intensified agricultural and industrial activities, certain reaches in the middle and lower Yiluo River have experienced varying degrees of nitrogen and phosphorus pollution, leading to increased water-quality fluctuations and growing pressure on aquatic ecological security. Scientifically and accurately characterizing water-quality variation is not only a key component of watershed water-resources management and environmental governance, but also an essential prerequisite for ecological protection and sustainable development.[1-2]

In the field of comprehensive river water-quality assessment, researchers have evaluated water-quality status and identified dominant pollution-contributing factors from multiple perspectives. In 2023, Ni Jian and colleagues proposed a water-quality assessment model integrating neural networks with an improved D–S evidence theory; validation based on five monitoring sections in southern Hebei indicated that the proposed model could improve assessment accuracy.[3] In 2024, Chen Yinghui and co-workers applied fuzzy comprehensive evaluation and the improved Nemerow pollution index to assess shallow groundwater in the Chaixuan Basin of Zhangjiakou, showing that overall groundwater quality met Class III standards, while some areas exhibited relatively poor quality. [4] In 2025, Fang Jiahui and colleagues evaluated the water quality of Qionghai Lake using an improved comprehensive water-quality identification index model based on game theory and a composite-weight variant of the same index. Their results suggested that the improved model better reflected the actual conditions of the study area and more effectively identified key environmental variables influencing water quality across different periods.[5]

Despite these advances, limitations remain in current studies on river water-quality assessment and its variation mechanisms. Conventional methods often rely on fixed thresholds and single, static weighting schemes, which makes it difficult to fully represent the inherent fuzziness and gradual transitions of water-quality states. Therefore, there is an urgent need to develop an integrated evaluation framework incorporating multiple and adaptive weighting strategies to address these gaps.

Accordingly, this study uses continuous monitoring data from the Qilipu section of the Yiluo River (2022–2024). Composite weights are established by combining the CRITIC method and the entropy weight method, and an improved fuzzy comprehensive evaluation approach is employed to investigate the dynamic evolution of water-quality grades.

2. Research Area and Data Sources

The Yiluo River flows through Shaanxi and Henan provinces, with a drainage area of 18,881 km^2. In this basin, the Yi River-historically known as the Luan River-has a main-channel length of 446.9 km and is the largest tributary of the Luo River. It originates from Sanhe Village, Taowan Town, on the southern slope of the Xiong'er Mountain in Luanchuan County, then flows through Song County, Yichuan County, and the urban area of Luoyang, and finally joins the Luo River at Yang Village, Guxian Town, Yanshi City. Another major tributary, the Luo River, is 265 km long, controls a catchment area of 6,041 km^2, and has a multi-year mean runoff of 1.296×10^9 m^3.

In this study, the Qilipu monitoring section of the Yiluo River in the Yellow River Basin was selected for water-quality assessment and prediction. Qilipu is a national control (state-controlled) section located in Zhengzhou, Henan Province, where real-time measurements are released every four hours. The data are publicly available through the National Real-Time Data Publishing System for Automatic Surface Water Quality Monitoring. Because routine station maintenance may result in occasional missing observations, the dataset contains missing values as well as anomalous records induced by external factors. Given the four-hour sampling frequency, the large sample size, and the relatively low proportion of missing data, a listwise deletion strategy (i.e., removing rows with missing values) was adopted to obtain a complete dataset for subsequent analysis.

The study period spans from January 1,2022,to December 31,2024,yielding a total of 6,177 observations. During this period, station maintenance occurred 52 times; after data cleaning, 6,125 valid records were retained for analysis.

3. Research Methods

All evaluation and prediction methods adopted in this study take the state-controlled Qilipu section of the Yiluo River as the research target. The improved fuzzy comprehensive evaluation method establishes indicator weights and membership functions based on the *Environmental Quality Standards for Surface Water* and the monitoring data from this section, while the GA–BP prediction model is developed using the time-series characteristics observed during 2022–2024.

3.1. Weights of Evaluation Indicators

3.1.1. CRITIC Weighting Method

The CRITIC method emphasizes the contrast intensity and inter-criterion conflict when assigning weights, effectively down-weighting highly correlated indicators and reducing information redundancy, thereby improving the scientific rigor and reliability of the comprehensive evaluation results.[6]

An m*n data matrix $X=(X_{ij})$ is constructed, where m denotes the number of objects to be evaluated and n denotes the corresponding evaluation indicators. Let $X=(X_{ij})$ be the (i, j)-th element of X. After indicator orientation (benefit-type) transformation, the processed element is denoted as $X_{ij}"$.

1) Sample selection

For benefit-type indicators (pH and DO):

$$X'=\frac{X_{ij}-\min(X_{ij})}{\max(X_{ij})-\min(X_{ij})}$$

Note that the CRITIC stage is used only to quantify indicator information content. In weight computation, pH is treated as a benefit-type indicator; however, the final water-quality class for pH is still determined according to the Environmental Quality Standards for Surface Water.

For cost-type indicators (CODMn, NH_3-N, and TP):

$$X'=\frac{\max(X_{ij})-X_{ij}}{\max(X_{ij})-\min(X_{ij})}$$

2) Determination of indicator contrast

The contrast among indicators is calculated as follows:

$$\delta_j=\sqrt{\frac{\sum_{i=1}^{n}(X_{ij}-\bar{X}_j)^2}{n-1}}$$

$$\bar{X}_j=\frac{1}{n}\sum_{i=1}^{n}X_{ij}$$

Where δ_j denotes the standard deviation of indicator j across the m samples.

3) Determination of inter-criterion conflict

The conflict among indicators is computed by:

$$R_j=\sum_{i=1}^{n}(1-r_{ij})$$

Where r_{ij} denotes the Pearson correlation coefficient between indicators i and j.

4) Determination of indicator information content

The magnitude of C_j determines the amount of information carried by indicator j and thus its contribution to the evaluation system:

$C_j = \delta_j R_j$

5) Calculation of indicator weights

$$W_j = \frac{C_j}{\sum_{j=1}^{n} C_j}$$

3.1.2. Entropy Weight Method

The entropy weight method is an objective weighting approach based on information entropy. It measures the entropy of each indicator across samples to reflect its degree of variation: the smaller the entropy and the larger the divergence coefficient, the more information the indicator provides and the larger its assigned weight . [7]The procedure is as follows:

1) Data selection

Select m indicators with a total of n samples. Let x_ij denote the value of indicator j for sample i.

2) Data normalization

For benefit-type indicators (pH and DO):

$$X' = \frac{X_{ij} - \min(X_{ij})}{\max(X_{ij}) - \min(X_{ij})}$$

For cost-type indicators (CODMn, NH_3-N, and TP):

$$X' = \frac{\max(X_{ij}) - X_{ij}}{\max(X_{ij}) - \min(X_{ij})}$$

Compute the proportion of sample i under indicator j:

$$P_{ij} = \frac{X_{ij}}{\sum_{i=1}^{n} X_{ij}}$$

Compute the information entropy of indicator j:

$$e_j = -\frac{1}{\ln(n)} \sum_{i=1}^{n} P_{ij} \ln(P_{ij})$$

where n is the number of samples.

Compute the divergence coefficient of indicator j:

$$d_j = 1 - e_j$$

3) Weight calculation
The weight of indicator j is calculated as:

$$W_j = \frac{d_j}{\sum_{j=1}^{m} W_j X_{ij}}$$

3.1.3. Composite Weights

Let μ denote the weights obtained by the CRITIC method and η denote the weights obtained by the entropy method. To obtain composite weights that are as close as possible to both μ and η without favoring either set, the minimum discrimination information principle is adopted. The composite weights are computed as:

$$W_i = \frac{\sqrt{\mu_i \eta_i}}{\sum_{i=1}^{j} \sqrt{\mu_i \eta_i}}$$

3.2. Improved Fuzzy Comprehensive Evaluation Method

Fuzzy comprehensive evaluation is widely used for water-environment quality assessment. Because water environment systems are influenced by many factors and involve complex mechanisms, this method introduces fuzzy mathematics and uses the Environmental Quality Standards for Surface Water as the assessment basis. Membership functions of each indicator to different water-quality classes are constructed from monitoring data, and indicator values are integrated with weights to produce an overall water-quality classification for the study area.

3.2.1. Establishment of the Factor Set and Evaluation Set

According to the National Environmental Monitoring Center, pH, dissolved oxygen (DO), permanganate index (CODMn), ammonia nitrogen (NH_3-N), and total phosphorus (TP) are selected as evaluation factors to form the factor set U. Based on the Environmental Quality Standards for Surface Water (GB 3838-2002), the water quality grading standard set V is established, where each element corresponds to a class (Class I to Class V)

3.2.2. Construction of the Fuzzy Weight Vector W

W is an m-dimensional row vector determined by allocating weights according to the contributions of pollution factors to environmental pollution and the synergistic/antagonistic effects among multiple factors. In this study, composite weights are derived by combining CRITIC and entropy weights to comprehensively account for information content, variability, and inter-criterion conflict among indicators. This yields a more objective, rational, and scientifically grounded weighting process and improves the accuracy of comprehensive water-quality evaluation.[8-9]

3.2.3. Membership Functions and Fuzzy Relation Matrix

A descending semi-trapezoidal membership function is used to define membership degrees. Dissolved oxygen is a “larger-is-better” indicator; therefore, the semi-trapezoidal function is designed in a reversed manner. pH is an “interval-optimal” indicator, whereas the remaining three indicators are “smaller-is-better” and use the standard descending semi-trapezoidal form. Introducing semi-trapezoidal membership functions enables a continuous representation of each indicator’s contribution to each class, making the results more consistent with the gradual transition characteristics of real water environments.[10-11]The membership functions can be expressed as follows:

When j = 1:

$$r_{ij} = \begin{cases} 0, C_i > \mathrm{S}_{i2} \\ \dfrac{S_{i2} - C_i}{S_{i2} - S_{i1}}, S_{i1} < C_i < S_{i2} \\ 1, C_i < \mathrm{S}_{i1} \end{cases}$$

When j = 2, 3, 4:

$$r_{ij} = \begin{cases} \dfrac{C_i - S_{i(j-1)}}{S_{ij} - S_{i(j-1)}}, S_{i(j-1)} < \mathrm{C}_i < \mathrm{S}_{i\mathrm{j}} \\ 1, C_i = S_{ij} \\ \dfrac{S_{i(j+1)} - C_i}{S_{i(j+1)} - S_{ij}}, S_{ij} < C_i < \mathrm{S}_{i(j+1)} \end{cases}$$

When j = 5:

$$r_{ij} = \begin{cases} 0, \mathrm{C}_i < \mathrm{S}_{i4} \\ \dfrac{C_i - S_{i4}}{S_{i5} - S_{i4}}, S_{i4} < C_i < S_{i5} \\ 1, C_i > \mathrm{S}_{i5} \end{cases}$$

where S_{ij} denotes the class-j standard (threshold) for factor i, and C_i denotes the measured concentration of pollutant i.

Construct the fuzzy relation matrix R:

$$R = \begin{bmatrix} r_{11} & r_{12} & \cdots & r_{1n} \\ r_{21} & r_{22} & \cdots & r_{2n} \\ \vdots & \vdots & & \vdots \\ r_{m1} & r_{m2} & \cdots & r_{mn} \end{bmatrix}$$

During 2022–2024, pH values remained within the optimal standard interval of 6.0–9.0. Therefore, pH does not differentiate Classes I–III in this study and is treated as a “constraint indicator” rather than a primary discriminating indicator.

3.2.4. Construction of the Fuzzy Comprehensive Evaluation Result Vector

Multiply the fuzzy weight vector W by the fuzzy relation matrix R to obtain the fuzzy comprehensive evaluation result B:

$$B = W \times R = (b_1, b_2, \cdots, b_n)$$

According to the maximum membership principle, if the maximum component of B corresponds to Class k, then the water-quality class of the evaluated object is determined as Class k.

4. Results and Discussion

4.1. Weights of Water-quality Parameters for Each Year (2022–2024)

The CRITIC weights, entropy weights, and composite weights for 2022, 2023, and 2024 were calculated using the equations in Section 3.1, as summarized in Table 1.

Table 1. Comprehensive Weight of Water Quality in the Yilu River

	2022			2023			2024		
	CRITIC weights	entropy weights	composite weights	CRITIC weights	entropy weights	composite weights	CRITIC weights	entropy weights	composite weights
pH	0.224	0.268	0.250	0.187	0.241	0.216	0.171	0.110	0.138
DO	0.161	0.186	0.177	0.222	0.331	0.276	0.189	0.189	0.190
COD_{Mn}	0.219	0.264	0.246	0.216	0.107	0.155	0.236	0.203	0.221
NH_3-N	0.137	0.177	0.159	0.175	0.131	0.154	0.242	0.275	0.260
TP	0.259	0.105	0.168	0.200	0.190	0.199	0.162	0.223	0.191

4.2. Fuzzy Relation Matrix

The fuzzy relation matrix calculated according to the method described in Section 3.2 is shown below.

Fuzzy relation matrix for 2022

$$R_1 = \begin{bmatrix} 1.000 & 1.000 & 1.000 & 0 & 0 \\ 1.000 & 0 & 0 & 0 & 0 \\ 0 & 0.994 & 0.006 & 0 & 0 \\ 0 & 0.676 & 0.324 & 0 & 0 \\ 0 & 0.968 & 0.032 & 0 & 0 \end{bmatrix}$$

Fuzzy relation matrix for 2023

$$R_2 = \begin{bmatrix} 1.000 & 1.000 & 1.000 & 0 & 0 \\ 1.000 & 0 & 0 & 0 & 0 \\ 0.439 & 0.561 & 0 & 0 & 0 \\ 0 & 0.930 & 0.070 & 0 & 0 \\ 0 & 0.880 & 0.120 & 0 & 0 \end{bmatrix}$$

Fuzzy relation matrix for 2024

$$R_3 = \begin{bmatrix} 1.000 & 1.000 & 1.000 & 0 & 0 \\ 1.000 & 0 & 0 & 0 & 0 \\ 0.582 & 0.418 & 0 & 0 & 0 \\ 0.150 & 0.850 & 0 & 0 & 0 \\ 0 & 0.717 & 0.283 & 0 & 0 \end{bmatrix}$$

By substituting the weight sets in Table 1 and the fuzzy relation matrices for 2022–2024 into the mathematical model in Section 3.2.4, the fuzzy comprehensive evaluation result vector B\mathbf{B}B for the Yiluo River water quality in 2022–2024 was obtained, as summarized in Table 2.

Table 2. Comprehensive Assessment Results of the Water Quality of the Yilu River

Year	I	II	III	Ⅳ	Ⅴ	Evaluation results
2022	0.427	0.765	0.308	0	0	II
2023	0.560	0.621	0.251	0	0	II
2024	0.496	0.588	0.192	0	0	II

According to the results in Table 2, the fuzzy comprehensive evaluation of water quality at the Qilipu section of the Yiluo River indicates that, after applying the fuzzy comprehensive evaluation to the state-controlled Qilipu section for 2022–2024, the water quality in all three years is classified as Class II based on the maximum membership principle. This suggests that the overall water quality at the study section remains at a relatively good level and meets the functional requirements for Class II waters specified in the *Environmental Quality Standards for Surface Water*.

From the perspective of composite-weight variation, the composite weights of ammonia nitrogen (NH_3–N) and total phosphorus (TP) generally increased or remained at relatively high levels during 2022–2024, implying that their constraining effects on the water-quality class gradually strengthened. These two parameters can therefore be regarded as key limiting factors preventing the water quality from further improving toward Class I.

In terms of the interannual trends in membership degrees, the study section exhibited a pattern of initial improvement followed by slight fluctuations. In 2023, the membership degree for Class I increased markedly to 0.560, compared with 0.427 in 2022, indicating that water-quality conditions shifted closer to the “excellent” class in that year; meanwhile, although the Class II membership degree decreased, it remained dominant. In 2024, the Class I membership degree decreased slightly to 0.496, while the Class II membership degree remained essentially stable, suggesting that the water quality experienced some fluctuations but showed no evident deterioration overall.

5. Summary

This study aimed to provide a more objective and realistic water-quality assessment for the Yiluo River by combining composite weighting (CRITIC–entropy) with an improved fuzzy comprehensive evaluation, under the assumption that this integration can better capture gradual class transitions than conventional fixed-weight approaches. Overall, the framework produced stable annual classifications for the state-controlled Qilipu section during 2022–2024 and supported interpretation of dominant limiting factors without relying on rigid threshold judgments.

The results indicate that the Qilipu section remained classified as Class II over the study period under the maximum membership principle, suggesting generally good water quality that meets Class II functional requirements. Changes in composite weights and membership trends further imply that NH_3–N and TP act as persistent constraints on further improvement toward Class I, providing a clear management implication for nutrient-oriented control and governance.

This work contributes a reproducible “standards–composite weights–membership functions” workflow that is suitable for high-frequency monitoring data and improves the objectivity and interpretability of multi-indicator assessment. Nonetheless, the study is limited by the use of a single monitoring section and a restricted indicator set, as well as assumptions embedded in membership-function design and the handling of missing data. Future research should expand to multiple sections, incorporate hydrometeorological and land-use drivers, apply more robust missing-data and uncertainty analyses, and benchmark the prediction module against stronger time-series models to verify generalizability.

In conclusion, the proposed framework offers a practical and scientifically grounded approach to diagnosing water-quality status and identifying key constraints, thereby supporting more targeted watershed management and ecological protection.

Conflicts of Interest

The authors declare that they have no conflict of interest.

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