ISSN: 2710-0588

Study on the Creep Model Considering Full-Process Damage in Rocks

Gang Li

School of Environment and Architecture, University of Shanghai for Science and Technology, Shanghai 200093, China

Abstract

Rock creep exhibits significant nonlinear characteristics, which become particularly pronounced during the accelerating creep stage under high shear stress, primarily due to the continuous accumulation of internal damage. The Nishihara model fails to adequately describe this phenomenon. To better characterize the full-stage creep process, this study introduces a damage variable into the component model based on damage mechanics theory and Lemaitre's strain equivalence principle, taking into account the full-process damage evolution. First, based on damage mechanics theory and the strain equivalence principle, a composite creep model consisting of a damaged generalized Kelvin model and a triggered nonlinear damage acceleration element is constructed. Finally, the Levenberg–Marquardt nonlinear least squares method is applied to fit selected experimental data, verifying the rationality of the theoretical model. The results show that the proposed full-process damage creep model can effectively describe the entire rock creep process, with the correlation coefficient between the fitted curves and experimental data exceeding 0.95.

Keywords

Shear Creep, Damage Creep, Nonlinearity, Damage Variable.

1. Introduction

The creep characteristics of rocks are essential indicators for evaluating the long-term stability of rock masses and provide a critical foundation for predicting the time-dependent deformation behavior of rocks [1]. With the rapid development of deep underground engineering in recent years, the issue of sustained high in-situ stress acting on underground structures has become increasingly prominent. Real-time monitoring of the evolution of the stress and strain fields in the surrounding rock after tunnel excavation, as they change over time, is crucial for ensuring construction safety, optimizing engineering design, and guiding long-term operations and maintenance. Therefore, a comprehensive understanding of the rock damage-creep mechanism and the establishment of a damage-creep model capable of describing the entire process of rock behavior is not only of significant theoretical importance but also provides a scientific basis for the long-term stability assessment of tunnels and the design of engineering support [2]. This approach is instrumental in advancing engineering practice in related fields.

Rock damage is a continuous process that begins in the early stages of loading and extends until failure. Under low shear stress, rocks generally experience only decaying creep, where the strain rate continually decreases. However, under sustained shear stress, as the shear modulus progressively decreases, damage accumulates through mechanisms such as the bonding failure of mineral particles and changes in pore structure. As shear stress increases, the rock enters a steady-state creep phase, where the strain rate stabilizes. This phase typically represents a dynamic equilibrium state in which new damage accumulates alongside the closure and adjustment of pre-existing damage, although the overall damage continues to increase at a slow

ISSN: 2710-0588

rate. Once the accelerated creep phase is reached, the strain rate increases, internal damage accumulation intensifies, and the rock ultimately reaches failure. Numerous domestic and international researchers have conducted extensive studies on rock rheological models, developing a series of classical damage models. These methods can generally be classified into two categories: the continuum damage mechanics approach, which is based on Kachanov's continuum theory and Lemaitre's strain equivalence principle, and describes damage evolution using internal variables; and the statistical damage approach, which uses probabilistic models such as the Weibull distribution to derive macroscopic damage behavior based on the statistical characteristics of microscopic defects.

Zhang Yibin et al. [3] improved upon existing models by introducing an unsteady element and incorporating a damage variable into the viscoplastic model, based on Kachanov's continuum theory, thereby accurately describing the accelerated creep phase. Liu Wenbo [4] applied statistical damage theory to modify the Western model, constructing a nonlinear creep damage model that effectively captures rock damage behavior. Wang Qihu et al. [5] proposed a fracture rock plastic deformation element to describe the instantaneous plastic deformation during rock creep, introducing an initial damage influencing factor, and developed a damage element that simulates rock accelerated creep. Liu et al. [6] integrated Kachanov's damage theory into the Bingham model by introducing a damage variable, proposing a nonlinear viscoelastic element to characterize the nonlinear behavior of the steady-state creep phase, and further incorporating a Western damage element to establish a novel nonlinear damage-creep model. Wang Junbao et al. [7], utilizing damage mechanics and Kachanov's damage theory, constructed an elastic element capable of reflecting the instantaneous deformation and accelerated creep deformation of rocks under aging damage.

In conclusion, while many scholars have introduced damage variables only in the accelerated creep phase, they have not fully captured the continuous evolution of rock behavior throughout the entire creep process. This is particularly important for rocks prone to damage, where the damage threshold is relatively low, and structural changes may occur during the decaying creep phase. Therefore, this paper proposes a comprehensive model that incorporates damage in the decaying creep phase, steady-state creep phase, and accelerated creep phase. By establishing a full-process rock damage-creep model, we aim to provide robust theoretical support for tunnel support and long-term stability prediction.

2. Establishment of the Damage Creep Model

Common rock creep models consist of elastic, viscous, and plastic elements. By combining these elements in series and parallel, it is possible to effectively describe rock creep behavior. In this study, the Western model, which is composed of a series combination of a generalized Kelvin body and a viscoplastic body, is selected. The generalized Kelvin body effectively captures both instantaneous and decaying creep, while nonlinear improvements to the components in the viscoplastic body allow for the description of the accelerated creep phase. Under the influence of shear stress levels and duration, the mechanical properties of rock gradually degrade. However, the model parameters of the elements in these conventional models are usually constant, which cannot simulate the degradation effect. To address this limitation, a damage variable is introduced into the model, transforming all the parameters into nonlinear variables. This approach enables a more accurate and effective representation of the rock creep process, accounting for the degradation behavior.

2.1. Damage Variable

The Lemaitre strain equivalence principle [8] is one of the most fundamental and widely used principles in damage models. It posits that the shear strain generated in damaged rock under

ISSN: 2710-0588

shear loading is equivalent to the shear strain generated in an imagined intact rock under effective shear stress.

$$\gamma = \frac{\tau}{G} = \frac{\tilde{\tau}}{\tilde{G}} \tag{1}$$

Here, τ represents the nominal shear stress, and σ_{eff}represents the effective shear stress. G is the shear modulus under the nominal shear stress acting on the rock, and G_{eff} is the shear modulus under the effective shear stress. The Rabotnov effective stress principle states that the effective shear stress σ_{eff} and the nominal shear stress τ are related by the following equation:

$$\tau = \frac{\tilde{\tau}}{(1-D)} \tag{2}$$

Substitute equation (2) into equation (1).

$$\gamma = \frac{\tilde{\tau}}{G(1-D)} = \frac{\tilde{\tau}}{\tilde{G}} \tag{3}$$

$$\tilde{G} = G(1-D) \tag{4}$$

That is,

$$D = 1 - \frac{\tilde{G}}{G} = 1 - \frac{G(t)}{G} \tag{5}$$

Zhang Qiangyong et al.[9] proposed that the rock damage variable changes with time in a negative exponential relationship. The damage evolution equation for this model is:

$$D(t,\tau) = \frac{G_0 - G_\infty}{G_0}(1 - e^{-\alpha(\tau)t}) \tag{6}$$

When $t = 0, D = 0$, and as $t \to \infty$, $D = \frac{G_0 - G_\infty}{G_0}$.

Where α is the shear stress damage coefficient.

According to the equivalent strain principle, it can be obtained that:

$$G(t) = G(1-D) \tag{7}$$

Some studies suggest that during the rock degradation process, the degradation pattern of the viscosity coefficient is similar to that of the shear modulus. We assume that both follow the same degradation pattern, thus:

$$\eta(t) = \eta(1-D) \tag{8}$$

Substitute equation (6) into equations (7) and (8), we get:

$$\begin{cases} G(t) = G\dfrac{G_\infty + (G_0 - G_\infty)e^{-\alpha(\tau)t}}{G_0} \\ \eta(t) = \eta\dfrac{G_\infty + (G_0 - G_\infty)e^{-\alpha(\tau)t}}{G_0} \end{cases} \tag{9}$$

2.2. Nonlinear Damage Model for the Accelerated Phase

During the accelerated phase, the creep rate of the rock increases rapidly, and the strain grows nonlinearly with time. Therefore, a nonlinear element is established, as shown in Figure 1. A power function is selected to perform a nonlinear modification on the viscous element [10], so that the improved nonlinear viscous element satisfies:

$$\eta = \begin{cases} \eta_2 & t < t_a \\ \eta_2(\dfrac{\gamma_a}{\gamma})^m & t \geq t_a \end{cases} \tag{10}$$

Where, when $t < t_a$, $m = 0$, and it degenerates to the basic element; when $t \geq t_a$, $m > 1$; γ_a represents the shear strain when entering the accelerated creep phase.

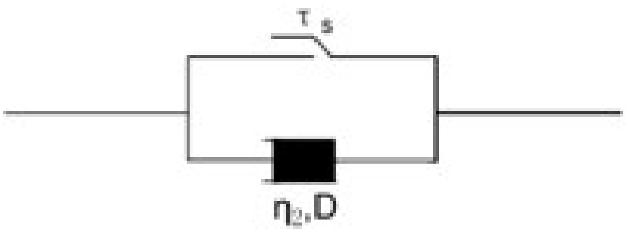

Figure 1. Nonlinear Damage Element

The nonlinear element is controlled by the critical shear stress τ_s. By introducing the damage variable (D) into the nonlinear element, its constitutive equation is:

$$\begin{cases} \tau = \tau_s + \eta_2(1-D)\dot{\gamma} & t < t_a \\ \tau = \tau_s + \eta_2(\dfrac{\gamma_a}{\gamma})^m(1-D)\dot{\gamma} & t \geq t_a \end{cases} \tag{11}$$

2.3. Damage Creep Model Equation

In summary, the full-process damage creep model constructed based on the rock sample's creep characteristics is shown in Figure 2. The model consists of three parts:γ_1 represents the damage elastic element, reflecting the instantaneous elastic strain; γ_2 represents the damage Kelvin body, reflecting the decay creep characteristics of the rock sample; and γ_3 represents the nonlinear damage model, which describes the steady-state creep when $t < t_a$ and the accelerated creep when $t \geq t_a$.

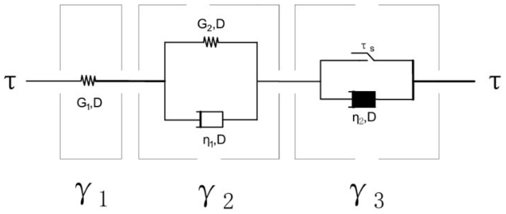

Figure 2. Full-Process Damage Creep Model

When $\tau<\tau_s$, the third part of the model does not function, and when $\tau\geq\tau_s$, the entire model is activated. According to the series and parallel relationship, the model equation is:

$$\begin{cases}\gamma=\gamma_1+\gamma_2,\tau=\tau_1=\tau_2 & \tau<\tau_s\\ \gamma=\gamma_1+\gamma_2+\gamma_3,\tau=\tau_1=\tau_2=\tau_3 & \tau\geqslant\tau_s\end{cases} \tag{12}$$

For the first part, its constitutive equation is:

$$\gamma_1=\frac{\tau_1}{G_1(1-D)} \tag{13}$$

Substituting equation (6) yields:

$$\gamma_1=\frac{\tau_1 e^{\alpha t}}{G_0-G_\infty+G_\infty e^{\alpha t}} \tag{14}$$

For the second part, its differential constitutive equation is:

$$G_2\gamma_2(1-D)+\eta_1(1-D)\dot{\gamma}_2=\tau_2 \tag{15}$$

Combining equation (6) and solving the differential equation, with γ=0 when t = 0 , we get:

$$\gamma_2=\frac{\tau_2}{G_2}\left(\frac{G_0 e^{\alpha t}}{G_0-G_\infty+G_\infty e^{\alpha t}}-e^{-\frac{G_2}{\eta_1}t}\right) \tag{16}$$

For the third part, its differential constitutive equation is:

$$\begin{cases}\tau_3=\tau_s+\eta_2(1-D)\dfrac{d\gamma_3}{dt} & t<t_a\\ \tau_3=\tau_s+\eta_2\dfrac{\gamma_a}{\gamma}(1-D)\dfrac{d\gamma_3}{dt} & t\geqslant t_a\end{cases} \tag{17}$$

Combining equation (6) and solving the differential equation, with $\gamma_3=0$ when t = 0, we get:

ISSN: 2710-0588

$\gamma_3=\gamma_a$, when $t=t_a=\frac{1}{\alpha}\ln\frac{G_0 e^{\frac{\alpha\gamma_a\eta_2 G_\infty}{G_0(\tau-\tau_s)}}-G_0+G_\infty}{G_\infty}$.

$$\gamma_3=\begin{cases}\frac{(\tau-\tau_s)G_0}{\alpha\eta_2 G_\infty}\ln(\frac{G_0-G_\infty+G_\infty e^{\alpha t}}{G_0}) & t<t_a\\ \gamma_a\left[(1-\lambda)\frac{(\tau-\tau_s)G_0}{\alpha\eta_2\gamma_a G_\infty}\ln(\frac{G_0-G_\infty+G_\infty e^{\alpha t}}{G_0})+\lambda\right]^{\frac{1}{1-\lambda}} & t\geq t_a\end{cases} \tag{18}$$

Based on the above analysis, when $\tau<\tau_s$, the nonlinear damage model is ineffective, and the damage creep model can describe the rock's stable creep behavior. When$\tau\geq\tau_s$, the damage creep model can describe the entire process of creep behavior. According to the superposition principle, the rock damage creep model equation is given by:

$$\begin{cases}\frac{\tau_1 e^{\alpha t}}{G_0-G_\infty+G_\infty e^{\alpha t}}+\frac{\tau_2}{G_2}(\frac{G_0 e^{\alpha t}}{G_0-G_\infty+G_\infty e^{\alpha t}}-e^{-\frac{G_2}{\eta_1}t}) & \tau<\tau_s\\ \frac{\tau_1 e^{\alpha t}}{G_0-G_\infty+G_\infty e^{\alpha t}}+\frac{\tau_2}{G_2}(\frac{G_0 e^{\alpha t}}{G_0-G_\infty+G_\infty e^{\alpha t}}-e^{-\frac{G_2}{\eta_1}t})+\frac{(\tau-\tau_s)G_0}{\alpha\eta_2 G_\infty}\ln(\frac{G_0-G_\infty+G_\infty e^{\alpha t}}{G_0}) & \tau\geq\tau_s,t<t_a\\ \frac{\tau_1 e^{\alpha t}}{G_0-G_\infty+G_\infty e^{\alpha t}}+\frac{\tau_2}{G_2}(\frac{G_0 e^{\alpha t}}{G_0-G_\infty+G_\infty e^{\alpha t}}-e^{-\frac{G_2}{\eta_1}t})+\gamma_a\left[(1-\lambda)\frac{(\tau-\tau_s)G_0}{\alpha\eta_2\gamma_a G_\infty}\ln(\frac{G_0-G_\infty+G_\infty e^{\alpha t}}{G_0})+\lambda\right]^{\frac{1}{1-\lambda}} & \tau\geq\tau_s,t\geq t_a\end{cases} \tag{19}$$

3. Model Parameter Inversion

Creep model parameter identification is essentially an optimization problem involving complex functions, and the reasonable determination of initial values for multi-parameter fitting is a significant challenge in practical applications. To address this, this study utilizes shear creep test data of mudstone [11] and employs the following method for parameter identification: First, the corresponding model (equation (19)) is constructed using a custom function in the nonlinear fitting platform of Origin software. Then, based on the Levenberg-Marquardt nonlinear least squares method, the experimental data is fitted to determine the model parameters. The results are shown in Table 1. After substituting the parameters obtained from Table 1 into the creep model (equation (19)), the comparison between the fitted curve and the experimental data is shown in Figures 3 and 4. The results indicate that the model established in this study can not only accurately describe the steady-state creep behavior of mudstone under low stress levels but also effectively characterize the non-steady-state creep characteristics of mudstone under high stress conditions. At different stress levels, the correlation coefficient (R^2) between the fitted curve and the experimental results remains above 0.98, further verifying the high reliability of the model.

Table 1. Model Fitting Parameters Table

σ/MPa	τ/MPa	G_1/MPa	G_2/MPa	η_1/(GPa·h)	η_2/(GPa·h)	α	m	R^2
0.2	0.1	0.09901	0.35999	0.3203		0.02263		0.998
	0.2	0.11111	0.1211	0.09712		0.02869		0.997
	0.3	0.12397	0.07515	0.04792		0.03689		0.996
	0.4	0.13408	0.0489	0.03273		0.04562		0.997
	0.5	0.14837	0.04818	0.01368	12.05995	0.12897	2.2178	0.987

ISSN: 2710-0588

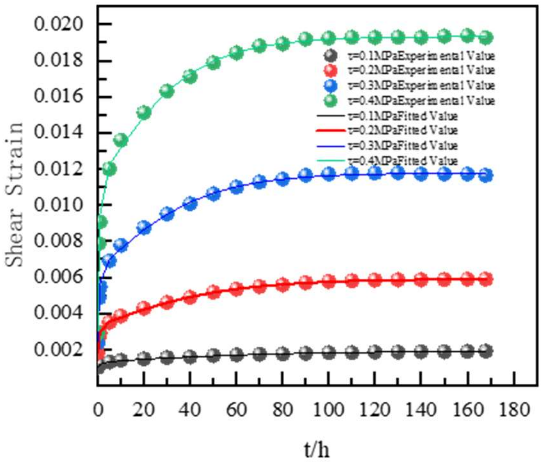

Figure 3. Experimental and Theoretical Values of Shear Stress 0.1-0.4 MPa

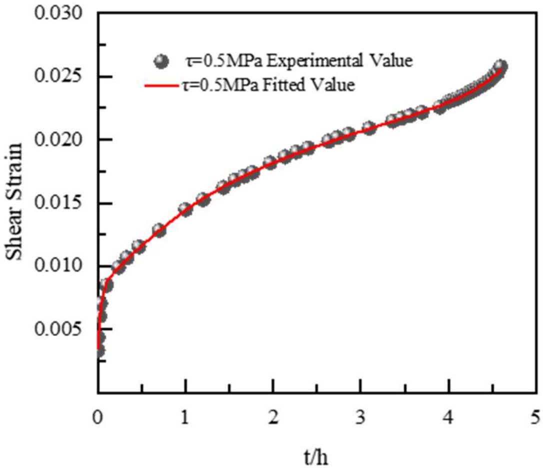

Figure 4. Experimental and Theoretical Values of Shear Stress 0.5 MPa

4. Conclusion

(1) To address the difficulty of the traditional Western model in capturing nonlinear accelerated creep under high shear stress, this paper, based on damage mechanics theory and the Lemaitre strain equivalence principle, introduces a damage variable into the component-based model. A shear damage creep model considering the entire damage evolution process is established, achieving a unified description of the decay, steady-state, and accelerated phases.

(2) In terms of damage variable construction, this paper establishes the relationship between nominal shear stress and effective shear stress based on the effective stress principle. Additionally, damage is introduced into parameters such as shear modulus and viscosity coefficient, transforming model parameters from constants to time-evolving nonlinear variables. This allows the model to reflect the essential characteristics of rock degradation over time.

(3) To capture the nonlinear characteristics of significantly increased creep rate during the accelerated phase, this paper constructs a triggered nonlinear damage acceleration element: when $t < t_a$, it degenerates into the basic element; when $t \geq t_a$ and shear stress reaches the critical value τ_s, the acceleration term participates in the response, enabling the model to distinguish between low-stress steady-state creep and high-stress full-process creep behavior.

(4) Based on mudstone shear creep test data, the Origin custom function is used in conjunction with the Levenberg–Marquardt nonlinear least squares method for parameter inversion. The

ISSN: 2710-0588

results indicate that the model accurately fits the steady-state creep curve at low stress levels and effectively characterizes non-steady-state creep behavior at high stress levels. The fitting correlation coefficient R^2 remains above 0.98 at different stress levels, confirming the model's reliability and applicability.

References

[1] Sun Jun. Some Advances in the Study of Rock Rheology and Its Engineering Applications. Journal of Rock Mechanics and Engineering, 2007, (06): 1081-1106.

[2] Shen Zhijun, Zhan Xianjun, Shang Haisong, et al. Long-Term Safety Monitoring Analysis of Tunnel Structure in Faulted Sections of Qiyueshan Tunnel. Modern Tunnel Technology, 2013, 50(02): 164-172. DOI:10.13807/j.cnki.mtt.2013.02.025.

[3] Zhang Yibin, Zhang Yihai, Ma Haitao, et al. Research on Rock Nonlinear Viscoelastic Damage Creep Model and Its Parameter Identification. China Safety Science Journal, 2023, 19(06): 13-19.

[4] Liu Wenbo, Zhang Shuguang, Chen Lei, et al. Research on Rock Accelerated Creep Model Based on Statistical Damage Theory. Journal of Geotechnical Engineering, 2020, 42(09): 1696-1704.

[5] Wang Qihu, Ye Yicheng, Liu Yanzhang, et al. Rock Creep Full-Process Constitutive Model Considering Initial Damage and Creep Damage. Rock and Soil Mechanics, 2016, 37(S1): 57-62. DOI:10.16285/j.rsm.2016.S1.007.

[6] Liu H Z, Xie H Q, He J D, et al. Nonlinear Creep Damage Constitutive Model for Soft Rocks. Mechanics of Time-dependent Materials, 2017, 21(1): 73-96.

[7] Wang Junbao, Liu Xinrong, Shao Zhushan, et al. Study on Rock Nonlinear Creep Damage Model. Modern Tunnel Technology, 2014, 51(03): 79-84. DOI:10.13807/j.cnki.mtt.2014.03.013.

[8] Voyiadjis G Z, Kattan P I. Damage Mechanics. [S.l.]: CRC Press, 2005: 114-128.

[9] Zhang Qiangyong, Yang Wendong, Zhang Jianguo, et al. Variable Parameter Creep Damage Constitutive Model and Its Engineering Application. Journal of Rock Mechanics and Engineering, 2009, 28(04): 732-739.

[10] Yu Bingbing, Li Qing, Zhao Tongde, et al. Full-Time Nonlinear Creep Damage Model for Rocks Based on Stress and Time Dual Thresholds. Journal of Rock Mechanics andEngineering, 2023, 42(08): 1928-1944. DOI:10.13722/j.cnki.jrme.2022.1147.

[11] Li Ning, Zhang Maojian, Zhang Huili, et al. Shear Creep Characteristics and Constitutive Model of Red Layer Mudstone in Central Yunnan. Journal of Central South University (Science and Technology Edition), 2024, 55(09): 3520-3529.