

A Review on Event-Triggered State Estimation for Time-Delay Neural Networks

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Abstract

In practical engineering applications, time delays are inherent in neural networks due to signal transmission lags, hardware response limits and network communication constraints, which may induce oscillation, chaos and even system instability. Meanwhile, limited measurable outputs and external noise make it impossible to obtain full neuron states directly, so state estimation has become an indispensable technique. However, traditional time-triggered transmission leads to massive redundant data and unnecessary communication consumption. Event-triggered mechanism (ETM) transmits data only when system states violate preset trigger conditions, which effectively reduces communication load while ensuring satisfactory estimation performance. This paper systematically reviews the research progress of event-triggered state estimation for time-delay neural networks. Firstly, basic concepts, system modeling and core problems are introduced. Then, key theoretical tools including Lyapunov-Krasovskii (L-K) functional, integral inequalities and stability analysis methods are summarized. Furthermore, typical event-triggered protocols, estimator design approaches and unified performance indices are elaborated. Finally, existing challenges and future research directions are discussed, aiming to provide a clear reference for researchers in related fields.

Keywords

Time-delay neural networks; State estimation; Event-triggered mechanism; Lyapunov-Krasovskii functional; Linear matrix inequality; Dissipativity.

1. Introduction

Neural networks have been widely applied in pattern recognition, signal processing, intelligent control, industrial automation and robotic systems due to their strong nonlinear approximation, self-learning and parallel information processing capabilities. In actual physical implementation, signal transmission among neurons, hardware response delays and network-induced lags make time delay an unavoidable inherent attribute. Time delays often degrade system dynamic performance, cause oscillation and chaos, and even lead to instability [1]. In addition, due to high interconnection of massive neurons, measurable outputs usually contain only partial state variables. Meanwhile, sensor measurements are inevitably interfered by external noise. Therefore, state estimation, which aims to reconstruct real system states from limited measurable data, has become a core issue in the analysis and design of time-delay neural networks.

With the rapid development of networked control systems, state estimation of neural networks is gradually implemented through wireless communication networks [2]. Limited bandwidth and communication resources make traditional time-triggered periodic sampling cause serious problems such as network congestion, packet loss and increased time delay, which seriously

reduce the reliability and accuracy of state estimation. As an effective on-demand transmission strategy, event-triggered mechanism breaks the periodic transmission mode. Data is transmitted only when the estimation error or output deviation exceeds a predefined threshold, which greatly reduces unnecessary data transmission and saves communication resources. Therefore, event-triggered state estimation for time-delay neural networks has become a cutting-edge cross-research field of control science, neural dynamics and network communication. In recent years, fruitful achievements have been made in event-triggered state estimation for time-delay neural networks. This paper focuses on summarizing the research status, core methods, typical performance indices and development trends in this field.

2. System Modeling and Problem Formulation

2.1. Time-Delay Neural Network Model

A general class of continuous-time time-delay neural networks is described as follows:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bf(x(t)) + Cf(x(t-d(t))) + D\omega(t) \\ y(t) = Ex(t) + Ff(x(t)) + Gf(x(t-d(t))) + H\omega(t) \\ z(t) = Lx(t) \end{cases}$$

where $x(t)$ is the neuron state vector; $y(t)$ is the measurable output; $z(t)$ is the signal to be estimated; $\omega(t)$ is external disturbance; $f(\cdot)$ is the activation function satisfying sector-bounded constraints; $d(t)$ is time-varying delay satisfying $0 \leq d(t) \leq h$, $|d(t)| \leq \mu$; $A, B, C, D, E, F, G, H, L$ are known parameter matrices with appropriate dimensions.

2.2. Event-Triggered Mechanism

Event-triggered mechanism determines the data transmission instants. Let t_k be the latest transmission instant. At sampling time t , the current measurement $y(t)$ is compared with the latest transmitted data $y(t_k)$. Data transmission is triggered if the following condition is satisfied:

$$\|y(t) - y(t_k)\|^2 > \sigma \|y(t)\|^2$$

where $\sigma > 0$ is the trigger threshold.

Adaptive event-triggered mechanism (AETM) dynamically adjusts the threshold according to real-time system states, which achieves a better balance between estimation performance and communication efficiency.

2.3. State Estimator and Error System

A widely used Luenberger-type state estimator is designed as:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bf(\hat{x}(t)) + Cf(\hat{x}(t-d(t))) + K(y(t) - \hat{y}(t))$$

where K is the estimator gain matrix to be designed. Define the estimation error $e(t) = x(t) - \hat{x}(t)$. Then the estimation error system can be derived. The goal of event-triggered state estimation is to design the trigger mechanism and estimator gain K such that the error system is asymptotically stable and satisfies prescribed performance indices.

3. Key Theoretical Tools

3.1. Lyapunov-Krasovskii (L-K) Functional

L-K functional is the most widely used tool for stability analysis of time-delay systems. To reduce conservatism, many improved structures have been proposed:

Mode-dependent switched L-K functional: designed for switched systems divided by delay and its derivative.

Delay-product-type L-K functional: introduces delay product terms to utilize delay information sufficiently [4].

Relaxed L-K functional: relaxes positive definiteness constraints via quadratic inequalities.

3.2. Integral Inequalities

Integral inequalities directly affect the estimation accuracy and conservatism of stability conditions [3]:

Jensen inequality and Wirtinger-based inequality: basic integral inequalities with certain conservatism.

Bessel-Legendre (B-L) inequality: higher precision than Wirtinger inequality.

Improved B-L inequality combined with reciprocally convex inequality: introduces quadratic delay information.

Free-matrix-based integral inequality: increases flexibility by introducing free weighting matrices.

3.3. Stability and Performance Analysis Methods

Linear Matrix Inequality (LMI): transforms stability conditions into solvable LMIs.

Average Dwell Time (ADT): applied to switched time-delay systems to ensure exponential stability [5].

S-Procedure: used to deal with activation function constraints; extended time-varying S-Procedure further reduces conservatism.

Dissipativity theory: provides a unified framework including H_∞ , passivity and L_2 - L_∞ performance.

4. Event-Triggered State Estimation Strategies

4.1. Typical Event-Triggered Protocols

1. Static event-triggered mechanism: fixed threshold, simple structure and easy implementation.

2. Adaptive event-triggered mechanism: dynamically adjusts threshold to reduce trigger times [6].

3. Memory-based adaptive event-triggered mechanism: uses historical state peak information to optimize trigger logic.

4. Distributed event-triggered protocol: suitable for large-scale networked neural networks.

4.2. Common Performance Indices

5. H_∞ performance: suppresses the influence of energy-bounded disturbances.

6. L_2 - L_∞ performance: constrains the peak value of estimation error [7].

7. **Extended dissipativity**: unifies H_∞ , passivity, (Q,S,R) -dissipativity.

8. **Finite-time boundedness**: ensures system states within a certain bound in a finite interval.

4.3. Representative Research Results

Switched system modeling method: divides the system into four modes based on delay and its derivative, reduces conservatism significantly.

Improved B-L inequality approach: fully uses quadratic delay information to derive less conservative conditions.

Adaptive event-triggered dissipative estimation: achieves efficient communication and generalized dissipativity.

Non-fragile state estimation: enhances robustness against estimator gain perturbations.

5. Challenges and Future Directions

5.1. Existing Challenges

How to further reduce the conservatism of stability conditions.

Co-design under complex network factors: packet dropout, signal quantization, cyber-attacks [8].

Exclusion of Zeno behavior and hardware-oriented implementation.

Distributed event-triggered estimation for large-scale neural networks.

5.2. Future Research Directions

Intelligent event-triggered mechanisms based on machine learning.

Fixed-time and prescribed-time state estimation.

Multi-objective co-design: security, fault-tolerance and resource efficiency.

Practical applications in robotic systems, smart grids and biomedical engineering.

Novel L-K functionals and high-precision integral inequalities.

6. Conclusion

Event-triggered state estimation for time-delay neural networks provides an effective approach to balance estimation performance and communication resource consumption. This paper reviews system modeling, event-triggered protocols, theoretical tools, estimator design and performance indices. With the development of networked intelligent systems, this field will face new demands and challenges. Continuous theoretical innovation and practical exploration will further promote the engineering application of neural network systems.

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