

# ETAS Spatio-temporal Model and Seismic Activity Analysis

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## Abstract

Conventional Coulomb transfer only considers the triggering effect of moderate and strong earthquakes, ignoring that every earthquake has the ability of self-triggering. Statistical model, epidemic aftershock sequence model (ETAS), represents the normal seismic activity in a region. The quietness and activity of seismic activity can be detected by the systematic deviation between seismic activity and prediction rate, which can be regarded as the trigger effect of previous events and has a strong correlation with the coseismic trigger of Coulomb fracture stress change caused by strong earthquake transfer. The simulation results of Wenchuan-Lushan-Jiuzhaigou earthquake sequence show that the earthquake is not instantaneous, but triggered by the accumulation of seismic activity caused by the weak value of Coulomb change, which is consistent with the fracture theory of concave-convex body.

## Keywords

Empirical statistical model; Seismic activity simulation; Earthquake calm; Coseismic stress triggering.

## 1. Introduction

The static Coulomb stress change of coseismic earthquake and the earthquake calming and triggering before the main earthquake have attracted much attention [1]. The detailed discussion of calming mechanism [2] and triggering mechanism [3] shows that stress is a reasonable driving force of seismic activity. Lushan earthquake is located in the strong trigger area of Wenchuan earthquake, while Jiuzhaigou earthquake is located in the strict stress shadow area. How is it unlocked? ETAS model is a point process model and the most widely used statistical model to describe earthquake catalogue. It assumes that the magnitude of each event has nothing to do with triggering its own offspring. Normal seismic activity is a Poisson process. Abnormal seismic activity, such as quietness or activity, is identified by the deviation from the fitted ETAS model, which represents whether the previous event has a triggering effect. It has been proved that the triggering and calming of seismic activity are consistent with the change of coseismic Coulomb stress, and the significant deviation between seismic activity and model can be used as a stress meter to further understand the change of regional stress field.

The Wenchuan earthquake sequence on May 12, 2008 is the strongest earthquake activity on the Longmenshan fault zone in recent years, and the subsequent Lushan earthquake occurred in its extended aftershock area. Lushan earthquake is located in the strong Coulomb trigger area, and Wenchuan earthquake seems to have no objection to the trigger of Lushan earthquake. Jiuzhaigou earthquake is only 300 kilometers away from Lushan earthquake. It is controversial whether Jiuzhaigou earthquake is triggered by Wenchuan earthquake or Lushan earthquake, because the aftershock area of Jiuzhaigou earthquake is basically located in the strong shadow area, but some disputes also point out that it is located in the weak trigger area [4]. In this paper, Zhuang [5] and other methods are adopted, and the R language code of Jllian[6] is used to simulate the seismic data in the study area.

## 2. Research Methods and Data

### 2.1. Research methods

ETAS model is a time-marked point process, and its distribution is uniquely determined by conditional intensity function  $\lambda(t, x, y, m | H_t)$ , and conditional intensity function  $\lambda(t, x, y, m | H_t)$  satisfies.

$$\lambda(t, x, y, m | H_t) dt dx dy dm \approx P\{X \cap [t, t + dt] \times [x, x + dx] \times [y, y + dy] \times [m, m + dm] \neq \emptyset | H_t\} \quad (1)$$

Where  $H_t = \{(t_i, x_i, y_i, m_i) \in X : t_i < t\}$  is the earthquake occurrence history up to time  $t$ . Therefore, the conditional strength of the spatio-temporal model can be written as:

$$\lambda(t, x, y) = \mu(x, y) + \sum_{\{k: t_k < t\}} \kappa(m_k) g(t - t_k) f(x - x_k, y - y_k; m_k) \quad (2)$$

Other forms are as follows:

$$\lambda(t, x, y, m) = s(m) \left[ \mu(x, y) + \sum_{\{k: t_k < t\}} \kappa(m_k) g(t - t_k) f(x - x_k, y - y_k; m_k) \right] \quad (3)$$

In the above equation,  $\mu(x, y)$  is a function of background intensity, assuming it is independent of time. Functions  $g(t)$ ,  $f(x, y; m_k)$  and  $s(m)$  are normalized response functions (i.e. probability density functions) of the occurrence time, location and magnitude of the descendants of ancestors with magnitude  $m_k$ , respectively. And a non-stationary Poisson process with intensity function  $\kappa(m_k) g(t - t_k) f(x - x_k, y - y_k; m_k) s(m)$  is excited by the  $k$ -th event.

The conditional strength of ETAS has a similar form, the only difference is:

$$f(x, y; m) = \frac{q-1}{\pi D^2 e^{\alpha(m-m_0)}} \left( 1 + \frac{x^2 + y^2}{D^2 e^{\alpha(m-m_0)}} \right)^{-q} \quad (4)$$

Equation (4) is Gaussian short-range attenuation, and

$$f(x, y; m) = \frac{q-1}{\pi D^2 e^{\gamma(m-m_0)}} \left( 1 + \frac{x^2 + y^2}{D^2 e^{\gamma(m-m_0)}} \right)^{-q} \quad (5)$$

is a long-range attenuation [7]. This model is adopted in the following research.

The effects of time boundary and space boundary need to be considered in earthquake simulation [8], so the events in the study area and the study period are the target events, and others are the supplementary events.

## 2.2. Research data

Earthquakes with a longitude of 100-108, a latitude of 28-36 and a depth of 0.1-50km were downloaded from China Seismological Network Center, and the magnitude was above 1.0. The earthquake records before 2008 were analog records. In October 2008, 30 monitoring stations were added in Sichuan and Yunnan, and the complete magnitude reached 1.3. However, considering the background catalog, the earthquake catalog from January 11, 1983 to December 7, 2025 was selected, so the complete magnitude was reduced to 3.3.

ETAS model is sensitive to the change of intensive seismic activity, and the area is required to be small enough, so that the requirement of uniform background rate can be met. A polygonal area is selected, which contains most of the earthquakes in the aftershock areas of Wenchuan earthquake, Lushan earthquake and Jiuzhaigou earthquake. As shown in Figure 1. The main shocks used are listed in Table 1.

**Table 1.** Focal mechanism of the main shock in the Wenchuan earthquake sequence

Longitude	Latitude	Depth	Strike1	Dip1	Slip1	Mw	Date	cutMw
103.62	31.31	14.9	204	71	-169	6.1	2008-05-12 11:11:5.2	level6
103.12	30.22	21.9	212	42	100	6.6	2013-04-20 00:02:53.1	level6
103.89	33.21	16.2	151	79	-8	6.5	2017-08-08 13:19:53.2	level6

## 3. Results and Analysis

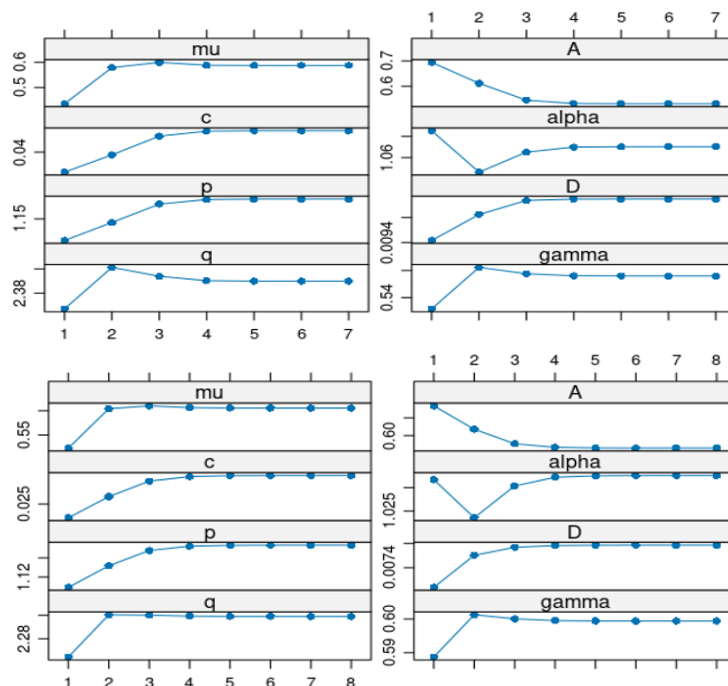
The polygon scope includes most aftershock areas of Wenchuan earthquake, Lushan earthquake and Jiuzhaigou earthquake. When fitting the earthquake catalogue within the polygon scope, two methods are adopted. The regional changes include and exclude Lushan earthquake respectively, in order to consider the contribution of Lushan earthquake to the region. For Wenchuan and Lushan earthquakes, the influence of Jiuzhaigou earthquake is considered.



**Figure 1.** Earthquake distribution characteristics in polygon study area of Wenchuan earthquake sequence

### 3.1. Wenchuan earthquake, including (excluding) Lushan earthquake, fitting results

Excluding Lushan earthquake (May 12, 2008-April 19, 2013), there are 897 total events, including 615 target events and 282 supplementary events (57 of which are located outside the study area and 225 events are located outside the study period). Figure 2 shows the fitting results of the parameters.

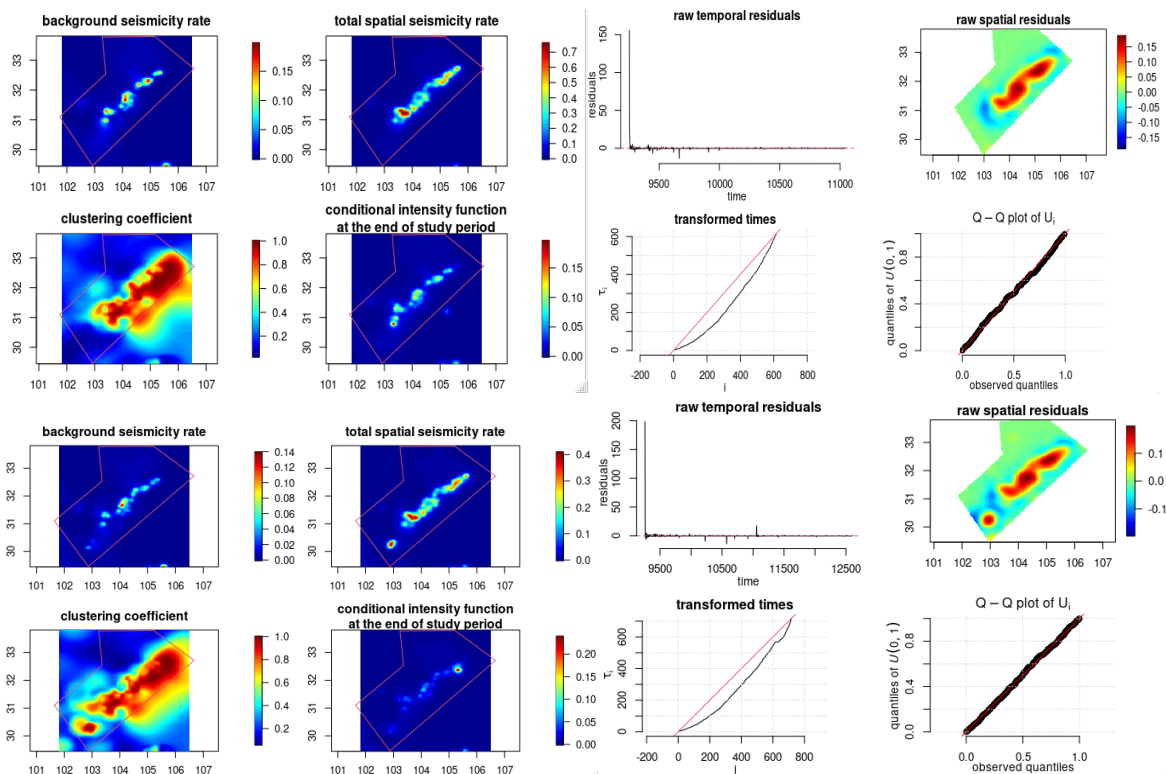


**Figure 2.** Parameter fitting results of Wenchuan earthquake, including (top) and excluding (bottom) Lushan earthquake.

Fig. 3 shows the simulation results of background seismic activity rate, total spatial seismic activity rate, cluster coefficient and conditional intensity function. The cluster coefficient indicates the intensity of aftershock activity, but the background seismic activity rate shows little clustering after the cluster earthquake is put forward from the total spatial activity. Conditional intensity represents the daily activity rate of the fitted earthquake in this area.

Time residuals show that the ETAS spatio-temporal model only considers the time feature, that is, the spatial density function is 1. It is found that there are residuals in both the time model and the spatio-temporal model, which shows that the parameters have some deviations from the perspective of model fitting, but the optimal parameters have been selected, so removing the possibility of model fitting differences can explain that the time deviation is the generation of earthquake calm, because the observation data after the conversion time obviously deviates from the model downward. The spatial residual is the largest in Longmenshan fault zone, and the fault classification characteristics may be the reason for this phenomenon.

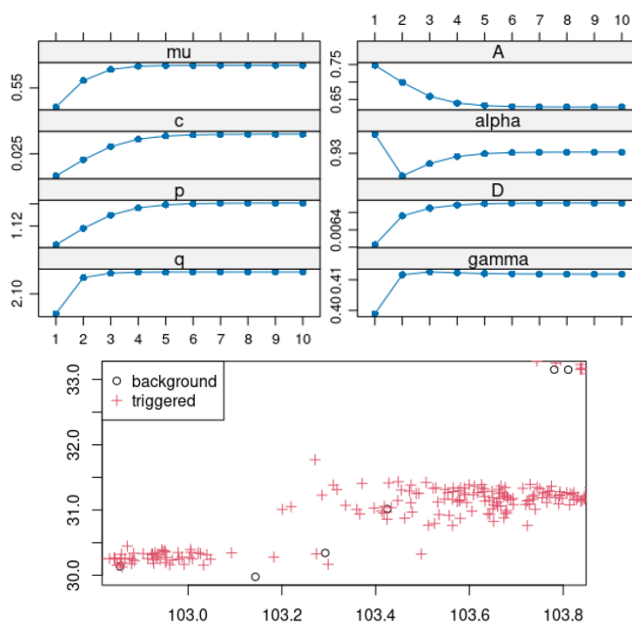
Including the fitting results of Lushan earthquake source (May 12, 2008-September 19, 2019), we can see that Lushan earthquake has made a certain contribution to the cluster coefficient, and the conditional intensity has weakened, which shows that Lushan earthquake has had a reverse effect on Wenchuan earthquake, that is, it has suppressed the aftershock area of Wenchuan earthquake. Although this force is very small compared with Wenchuan earthquake, it is worth noting that it has changed the northern part of Longmenshan fault.



**Figure 3.** Wenchuan earthquake, including (excluding) fitting results and fitting residuals of background rate, spatial seismic activity rate, cluster coefficient and conditional intensity function of earthquake catalogue with Lushan earthquake as the main trigger source.

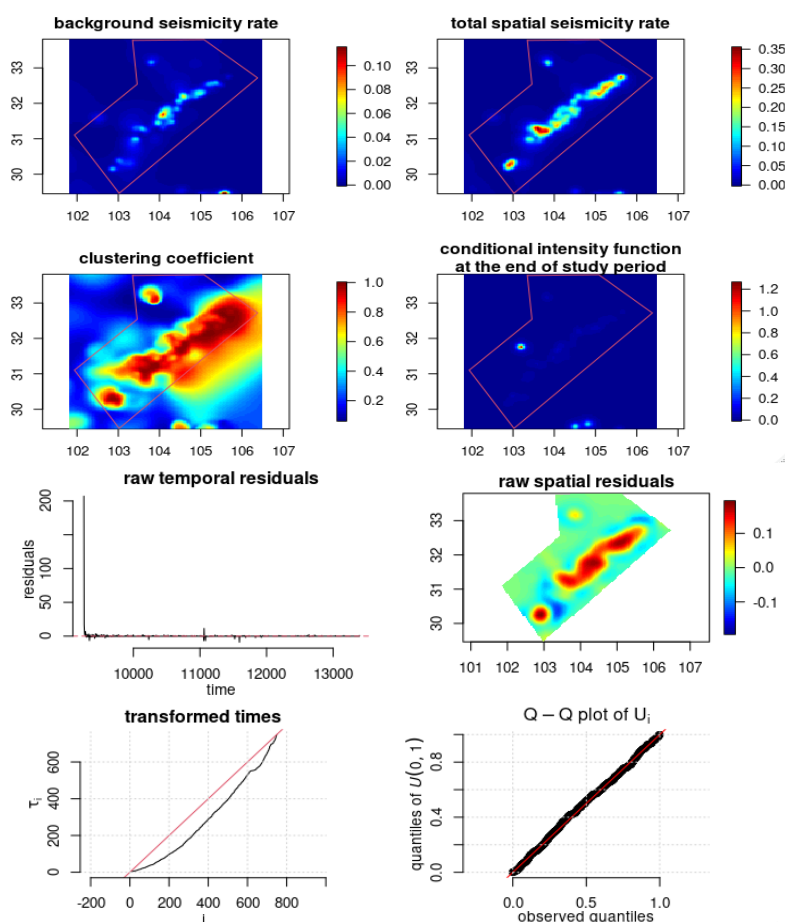
### 3.2. Fitting results of Jiuzhaigou earthquake

Figure 4 shows the fitting parameters including Jiuzhaigou earthquake, and the parameters have achieved good convergence in 10 times. A few background earthquakes triggered a large number of earthquakes.



**Figure 4.** Fitting parameter results of Jiuzhaigou earthquake (top) and obtained background earthquake and trigger events (bottom)

Fig. 5 shows the fitting parameters and seismic activity characteristics including Jiuzhaigou earthquake. The seismic background activity with complete magnitude of 3.5 has produced a large number of triggering events. After joining the Jiuzhaigou earthquake, a weak cluster appeared in the west of Longmenshan fault, which is located in the middle zone between Longmenshan fault and Jiuzhaigou earthquake, indicating that there may be a small broken fault between Jiuzhaigou earthquake and Longmenshan fault. Fig. 4 shows the fitting time residuals and space residuals, and the QQ diagram describing the model normality. The model after conversion time is displayed as a red straight line with slope, and the black line is the observed value. It shows that the occurrence of earthquake calm, Lushan earthquake will recover temporarily and the seismic activity will increase, but it is far from the level before Wenchuan earthquake. Before the Jiuzhaigou earthquake, the seismic activity increased obviously. Visual spatial residual found that Lushan earthquake shifted the spatial residual.



**Figure 5.** Fitting results and spatio-temporal residuals of Jiuzhaigou earthquake including Lushan earthquake.

#### 4. Discussion and Conclusion

The polygon study area covers the vast majority of earthquakes in the aftershock area of three main earthquakes, including the trigger area of Lushan earthquake and the Coulomb stress shadow area of Jiuzhaigou earthquake. Most of the source areas of Lushan earthquake are located in the aftershock area of Wenchuan, which can even be regarded as the aftershock of Wenchuan earthquake. Considering Wenchuan earthquake alone to fit the catalog, the time residual shows the occurrence of peace. Adding Lushan earthquake to the research target period, the fitting results show that Lushan earthquake has a certain contribution to the cluster coefficient. Although it is very small, it is worth noting that it weakens the fault zone in the north

of Wenchuan earthquake to a certain extent, and jumps the fractured section of Wenchuan earthquake in the middle. Assuming that some fractured sections have certain intensity, Coulomb stress plays a reverse role in Wenchuan.

Wenchuan earthquake put negative 20Mpa Coulomb stress on the focal area of Jiuzhaigou earthquake, and the occurrence of Lushan earthquake enhanced the cluster coefficient of Jiuzhaigou, which means that Lushan earthquake, which is 300 kilometers away, became the trigger source of Jiuzhaigou, although its intensity was much weaker than the shadow of Wenchuan earthquake, which was not found in previous studies.

The growing cluster in the middle of Longmenshan fault where Jiuzhaigou and Wenchuan earthquakes are located may be a small fracture zone, which is supposed to be the trigger factor of Jiuzhaigou. The increase of cluster coefficient caused by Lushan earthquake may reflect the retreat of some Coulomb stress shadows, and the adjustment of local stress prompted Jiuzhaigou earthquake.

## References

- [1] Ogata, Y., Detection of anomalous seismicity as a stress change sensor. *Journal of Geophysical Research: Solid Earth*, 2005. 110(B5).
- [2] Scholz, C.H., The critical slip distance for seismic faulting. *Nature*, 1988. 336(6201): p. 761-763.
- [3] Dieterich, J., V. Cayol and P. Okubo, The use of earthquake rate changes as a stress meter at Kilauea volcano. *Nature*, 2000. 408(6811): p. 457-460.
- [4] Junhua, et al., A study of the Coulomb stress and seismicity rate changes induced by the 2008 Mw 7.9 Wenchuan earthquake, SW China. *Journal of Asian Earth Sciences*, 2017.
- [5] Zhuang, J., Y. Ogata and D. Vere-Jones, Stochastic Declustering of Space-Time Earthquake Occurrences. *Journal of the American Statistical Association*, 2002. 97: p. 369-380.
- [6] Jalilian, A., ETAS: An R Package for Fitting the Space-Time ETAS Model to Earthquake Data. *Journal of Statistical Software*, 2019.
- [7] Zhuang, J. and D. Harte, Theme V – Models and Techniques for Analyzing Seismicity Basic models of seismicity: Spatiotemporal models. 2011.
- [8] Zhuang, J. and S. Touati. Theme V-Models and Techniques for Analyzing Seismicity Stochastic simulation of earthquake catalogs. 2015.