

Implementation of A Liquefaction Subroutine in ABAQUS Based on The Modified Hardin-drnevich Soil Backbone Curve Model

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Abstract

Liquefaction caused by earthquakes can affect the stability of underground structures. Numerical simulation of liquefaction is therefore necessary. By combining the modified Hardin-Drnevich soil constitutive model with Byrne's modified Martin pore pressure increment model, this paper develops an ABAQUS subroutine incorporating the modified Hardin-Drnevich soil constitutive model and the Byrne pore pressure increment model based on the implicit interface of ABAQUS. Numerical models in ABAQUS and FLAC3D are established respectively based on the undrained dynamic triaxial test results of Nanjing saturated fine sand, and the effective analytical capability of the proposed subroutine is preliminarily verified. Through comparing a total of 6 groups of numerical calculation results under different working conditions, this study verifies the influences of pore pressure growth trend, initial effective pore pressure and dynamic shear ratio on the liquefaction process. It is proved that the calculation results of the proposed subroutine in dynamic triaxial tests are superior to those of FLAC3D. The causes of numerical calculation errors are discussed, and the feasibility of developing the liquefaction numerical simulation subroutine for ABAQUS is verified.

Keywords

Numerical simulation; Byrne incremental model; Modified Hardin-Drnevich; Liquefaction.

1. Introduction

When seismic forces act on saturated sandy soil or sandy silt soil foundations, the soil undergoes volume compression due to vibration and tends to become denser, resulting in a sharp increase in pore water pressure. As the pore water pressure increases, the effective stress borne by the soil skeleton gradually decreases; once the effective stress is completely lost, the soil will lose its shear strength and eventually cause structural damage. At this time, the soil exhibits a liquid-like flow state, which is called soil liquefaction. Currently, various engineering projects are constructed in liquefiable areas, such as the Cao Fei Dan Industrial Zone in Tangshan [1], the main urban area of Tianjin [2], the banks of the Yangtze River [3], and the comprehensive tunnels in Hangzhou [4], all of which are located in potential liquefaction zones. Therefore, the research on liquefaction-related disasters in the site is quite important.

Specific experiments are often costly, but with the continuous improvement of people's chip manufacturing level and the increasing computing power of computers, numerical simulations conducted through computers are often accurate and cost-effective, which is conducive to improving the speed of liquefaction research. Currently, there are models that describe the increment of soil liquefaction [5]. At the same time, there have been finite element software that combine the liquefaction model with numerical simulation, providing references for many scholars to study sand soil liquefaction. Such as: SWANDYNE II [6], FLAC2D [7], FLAC3D [8],

DBLEAVES [9] and other software. However, some of these software are difficult to obtain, and some have overly complicated user graphical operation interfaces.

ABAQUS, as a finite element analysis software, has advantages such as rich functions, intuitive display, and good user graphic interaction design. At the same time, the software provides interfaces allowing users to run various subroutines such as their own-written material constitutive models. This makes up for the lack of relevant models in its program library. This paper will conduct secondary development of ABAQUS based on the Byrne pore pressure increment model and the modified Hardin-Drnevich soil skeleton curve to supplement the problem that ABAQUS itself cannot directly simulate soil liquefaction. The preliminary verification of the feasibility of ABAQUS' numerical simulation of the pore pressure increment model has been carried out.

2. Establishment of the Liquefaction Subroutine

2.1. Byrne pore pressure increment model

Based on the compatibility condition of the volume change of saturated soil during an earthquake, Martin et al. [10] established a basic formula for determining the increment of the saturated soil's pore water pressure. If the volume change of the soil is positive (indicating compression) and the volume change of the pore water is positive (indicating discharge), then:

$$\Delta\varepsilon_d + \Delta\varepsilon_r = \Delta\varepsilon_f \quad (1)$$

Among them, $\Delta\varepsilon_d$, $\Delta\varepsilon_r$ and $\Delta\varepsilon_f$ represent the volume compression of the soil, the change in soil volume with the decrease of effective normal stress, and the amount of pore water discharge, respectively.

If no drainage is performed $\Delta\varepsilon_f = 0$, then:

$$\Delta\varepsilon_d = -\Delta\varepsilon_r \quad (2)$$

Let E_r represent the resilient modulus of saturated sandy soil, then:

$$\Delta\varepsilon_r = -\Delta u/E_r \quad (3)$$

From equations (2) and (3), we can obtain:

$$\Delta u = E_r \Delta\varepsilon_d \quad (4)$$

Byrne [11] reviewed the relevant literature [12] and derived the Byrne simplified model of the Martin-Finn liquefaction model. The empirical relationship curve calculation formula for $\Delta\varepsilon_d$ and ε_d was obtained:

$$\Delta\varepsilon_d/\gamma_s = C_1 \exp(-C_2 \frac{\varepsilon_d}{\gamma_s}) \quad (5)$$

Among them, γ_s represents the amplitude of shear strain, $\Delta\varepsilon_d$ and ε_d are the incremental and cumulative soil strains respectively, and C_1 , C_2 are the test parameters.

Byrne [13] utilized specific experimental data to conduct research on C_1 and C_2 in the formula, and summarized the following empirical formula:

$$C_1 = 7600(D_r)^{-2.5} \quad (6)$$

$$C_2 = \frac{0.4}{C_1} \quad (7)$$

D_r represents the relative density of the soil, which can be calculated from the number of standard penetration blows.

The resilient modulus E_r of saturated sandy soil is determined through uniaxial compression tests:

$$E_r = \frac{(\sigma_{v0} - u)^{1-m}}{mk(\sigma_{v0})^{n-m}} \quad (8)$$

Among them, σ_{v0} represents the initial effective normal stress, while k , n , and m are the test parameters.

2.2. Development of UMAT Implicit Subroutines in ABAQUS

The ABAQUS software can use the FORTRAN language to write subroutines to modify multiple modules in the software. The subroutines written in this article use the UMAT interface to modify the calculation process related to the dynamic implicit method. The main process of writing UMAT is as follows: 1. Obtain the relevant results of the previous step calculation and the constitutive data. 2. Return the stress and Jacobian matrix through the user-defined subroutine. Before simulating the change in pore pressure of the soil mass, it is necessary to obtain relevant data such as effective stress. Therefore, this article divides the liquefaction subroutine into static and dynamic parts. The static part uses the general static calculation and eliminates the strain through the stress field balance.

Compared with other soil constitutive models, the modified Hardin-Drnevich can better simulate the test backbone curve and is insensitive to the test data [14]. Therefore, this article selects the modified Hardin-Drnevich soil backbone curve for subroutine development:

$$\tau = \frac{G_{max}\gamma}{1+(\gamma/\gamma_r)^a} \quad (9)$$

Here, τ represents shear stress; γ denotes shear strain; G_{max} is the initial maximum shear modulus; γ_r is the reference shear strain; and a is the test parameter.

Construct the hysteretic curves of different soil skeleton models under the action of irregular back-and-forth stress, in order to simulate the energy loss of the soil under seismic action. Use the extended Masing law to construct the hysteretic curves.

$$\tau - \tau_c = \frac{G_{max}(\gamma - \gamma_c)}{1+(\frac{\gamma - \gamma_c}{2\gamma_r})^a} \quad (10)$$

Here, τ_c and γ_c respectively represent the shear stress amplitude and shear strain amplitude at the starting point of unloading and reloading.

The increase in pore pressure due to soil vibration will have an impact on the dynamic properties of the soil. To achieve numerical simulation, it is necessary to establish the

relationship between vibration pore pressure and soil modulus. For less cohesive liquefiable soil, it can be assumed that the degradation is caused by the increase in vibration pore pressure. Therefore, the soil degradation can be described using the reduction of soil parameters due to the increase in vibration pore pressure. The G_{max} after degradation is:

$$G_{max}^* = G_{max}(1 - u)^n \quad (11)$$

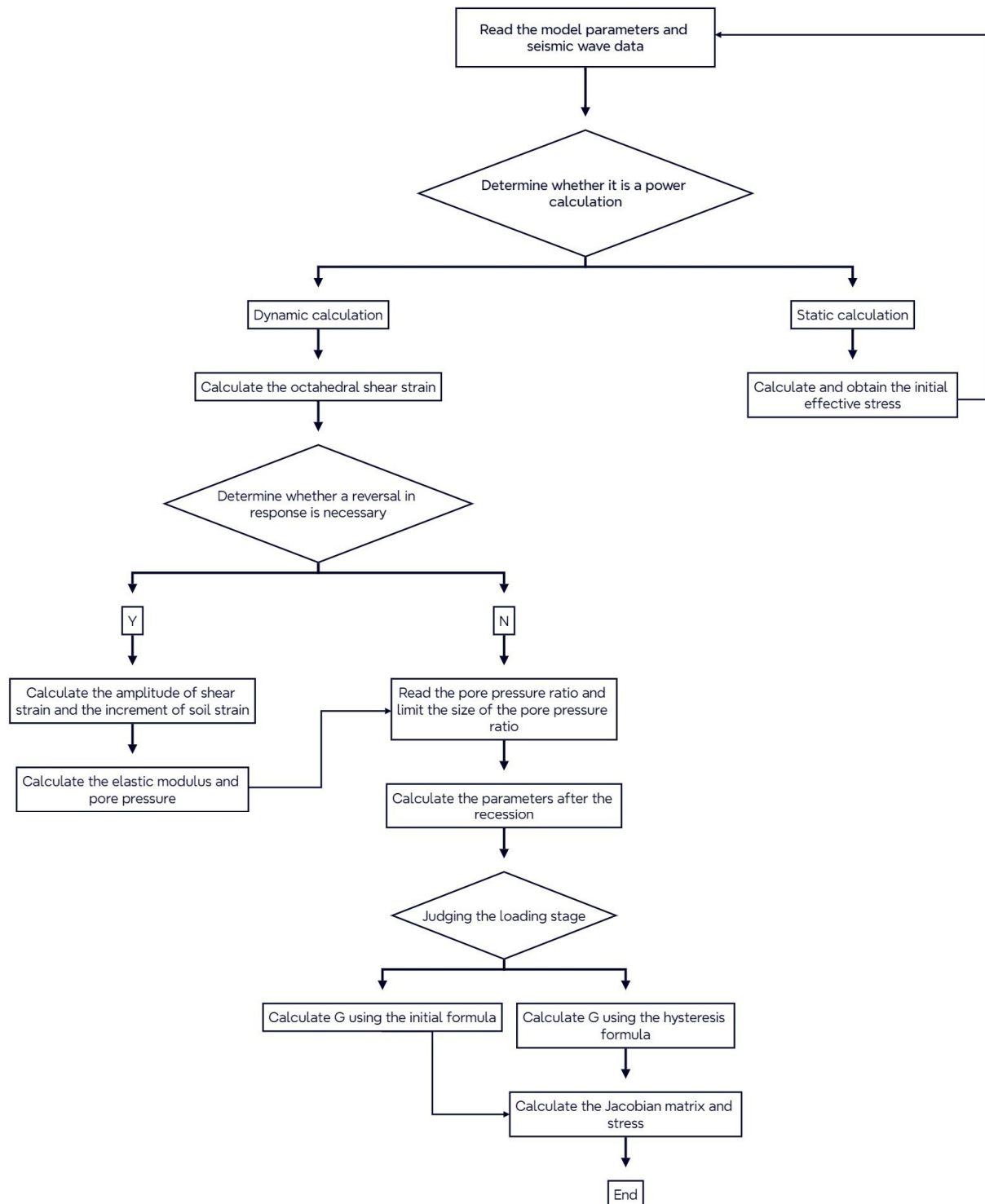


Figure 1. Subroutine calculation process

Among them, G_{max}^* represents the G_{max} value after the decline, and n is the test parameter, which is typically set to 0.5 [15].

Using G_{max}^* to calculate the γ_r after the decline, which is γ_r^* , and substituting it into the main curve, we obtain the main curve after the decline:

$$\tau = \frac{G_{max}^* \gamma}{1 + (\gamma/\gamma_r^*)^a} \quad (12)$$

After saturation of sandy soil liquefaction, it still retains a certain degree of strength. Additionally, to prevent the occurrence of infinite values in the subroutine calculations, the maximum pore pressure ratio is set at 0.99.

Fig. 1 is the specific calculation process of the subroutine: The model first reads the required parameters and determines the static and dynamic analysis steps. In the static analysis step, the effective confining pressure is obtained. In the dynamic analysis step, the octahedral shear strain is calculated and participates in the subsequent calculations. It is determined whether to reverse the calculation of pore pressure. Finally, different soil constitutive return stress and Jacobian matrices are used in different loading stages.

3. Subroutine Verification

3.1. Determination of relevant parameters for ABAQUS

This paper uses the undrained dynamic triaxial test data related to the saturated fine sand in Nanjing in the article [16], and uses MATLAB to fit and determine the relevant parameters of the modified Hardin-Drnevich constitutive curve. The fitting image is shown in Fig. 2. The fitting results of the three variables γ_r , G_{max} , and a are 0.0001, 79510000 Pa, and 0.9235 respectively.

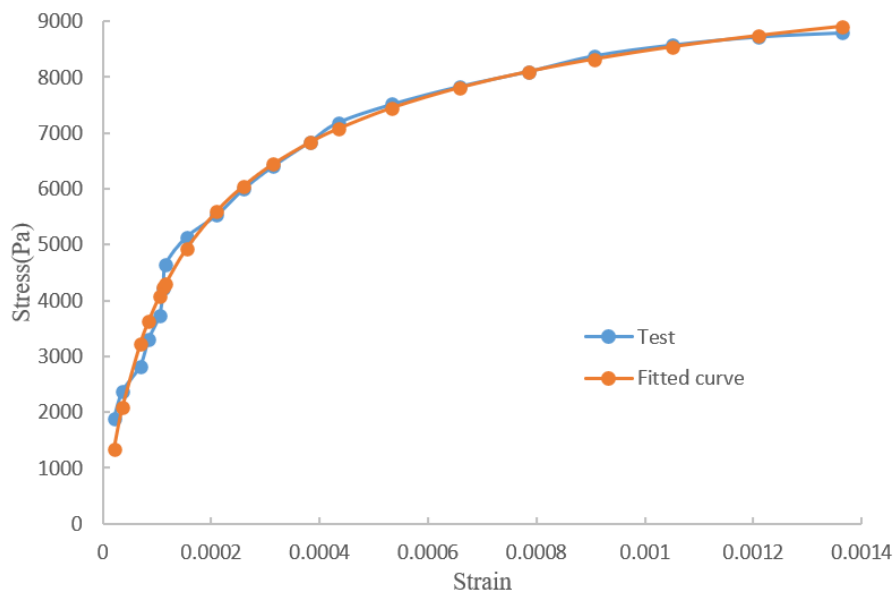


Figure 2. Fitted curve

The parameters of the Byrne pore pressure increment model were taken from Nanjing sand [17] for the relevant parameters of the saturated fine sand in Nanjing. The soil parameters of the subroutine are shown in the Table 1.

Table 1. Soil parameters

Model	Parameter	Value
Soil constitutive model	$G_{max}(\text{Pa})$	79510000
	γ_r	0.0001
	a	0.9235
	Stationary Poisson ratio	0.444
	Dynamic Poisson ratio	0.49
	Reduction of test parameters	0.5
Liquefaction model	C1	0.73
	C2	0.475
	K	0.0025
	N	0.62
	M	0.43

3.2. ABAQUS dynamic triaxial test numerical simulation

The Nanjing fine sand triaxial test was selected as the simulation object to verify the effectiveness of the subroutine in simulating the site liquefaction. The sample was composed of a 4cm-wide and 8cm-high soil body and two rigid blocks at the top and bottom. The rigid blocks were constrained rigidly and in contact with the soil body. The bottom rigid block was completely fixed, and the soil body and the top rigid block were fixed to prevent x-axis and z-axis displacements. The meshing was carried out using the C3D8 element, which is an eight-node linear hexahedral element. The global size of the mesh was set at 0.005m. The model assembly and meshing are shown in the following Fig. 3 and Fig. 4:

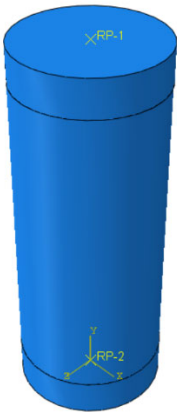


Figure 3. Model assembly

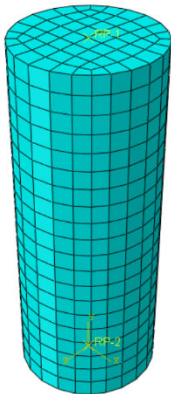


Figure 4. Model meshing

3.3. **FLAC3D dynamic triaxial test numerical simulation**

FLAC3D is a commercial numerical simulation software for geotechnical engineering. It comes with a module for calculating soil liquefaction. This paper uses the liquefaction module of FLAC3D to conduct cross-validation of subroutines. In FLAC3D, for dynamic fluid-solid coupling calculations, an explicit calculation method is used. This calculation method has relatively low efficiency. Additionally, for soil in the dynamic triaxial test within 10 cm, the calculation efficiency is further reduced. Therefore, to improve the calculation efficiency, this paper selects a single integration element for calculation. The soil constitutive model adopts the Finn model and Byrne mode. The relevant parameters are shown in the following Table 2.

Table 2. FLAC3D Soil parameters

Model	Parameter	Value
Soil constitutive model	Density (kg/m3)	2161
	Shear modulus (pa)	91.5e6
	Bulk modulus (pa)	786e6
	Internal friction angle	35
	Porosity	0.3
	Liquid density (kg/m3)	1000
	Liquid modulus (pa)	2e9
Liquefaction model	C1	0.73
	C2	0.475
	C3	0

The model is restricted in the z-axis displacement at the bottom, an inward confining pressure is applied around it, and a sinusoidal displacement is applied at the top. The damping ratio is 0.05 and the frequency is 20Hz. Here, the stress boundary condition at the top in the dynamic triaxial test is converted into the corresponding displacement boundary condition. The converted data is shown in the following Table 3.

Table 3. Stress-strain conversion

Stress	Strain
100kp0.15	8.3
100kp0.18	10
100kp0.21	11.7
50kp0.18	5
150kp0.18	14
150kp0.21	17

3.4. **ABAQUS and FLAC3D calculation results**

Numerical simulations were conducted for a total of 6 groups of the dynamic triaxial tests. Fig. 5-10 are the simulation results:

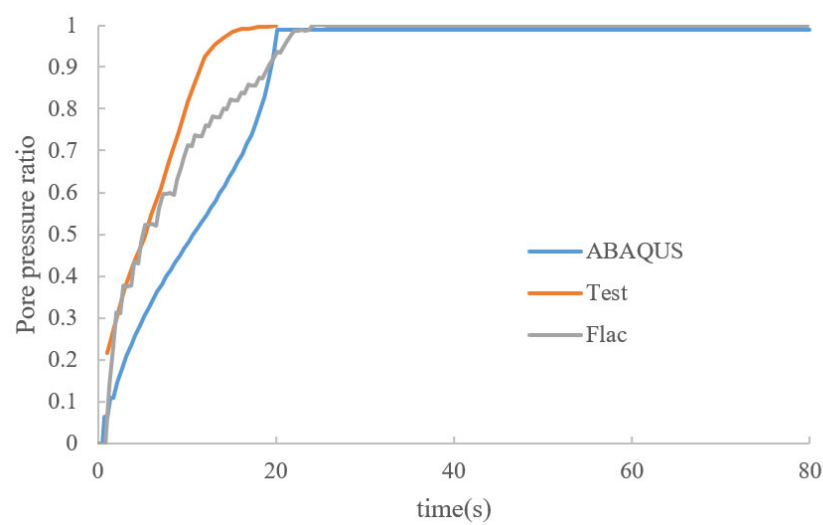


Figure 5. Dynamic triaxial tests 50kp0.18

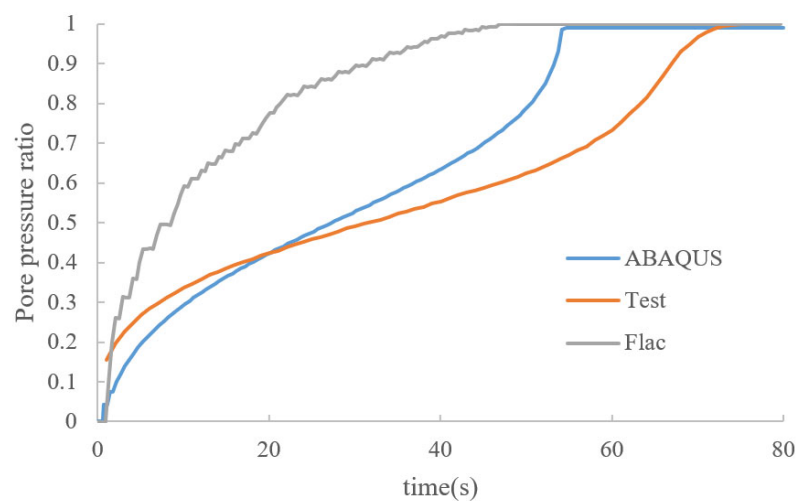


Figure 6. Dynamic triaxial tests 100kp0.15

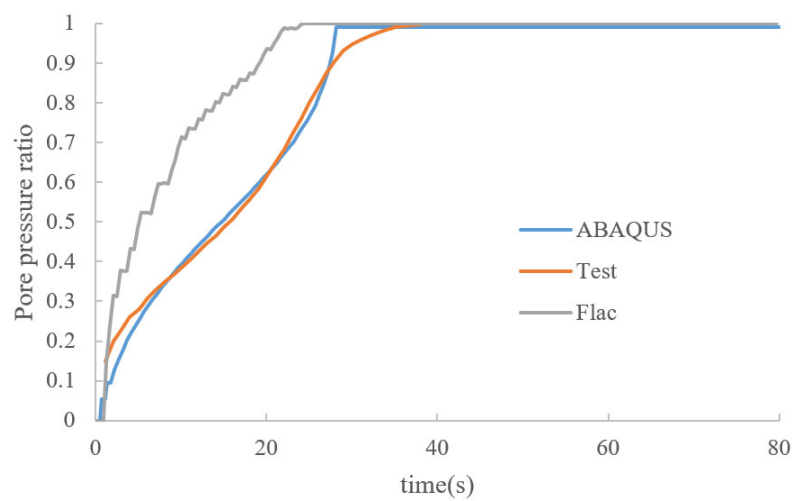


Figure 7. Dynamic triaxial tests 100kp0.18

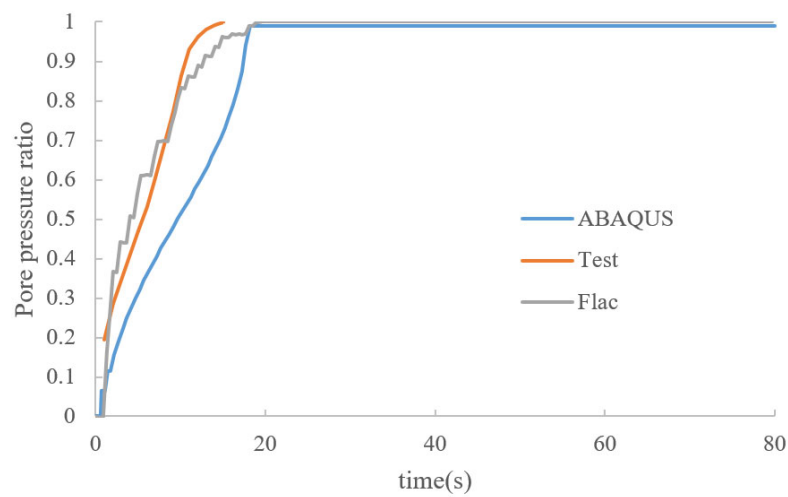


Figure 8. Dynamic triaxial tests 100kp0.21

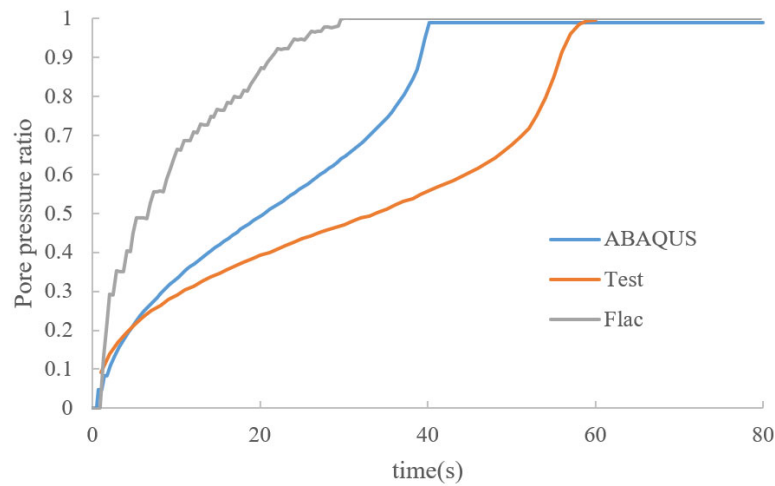


Figure 9. Dynamic triaxial tests 150kp0.18

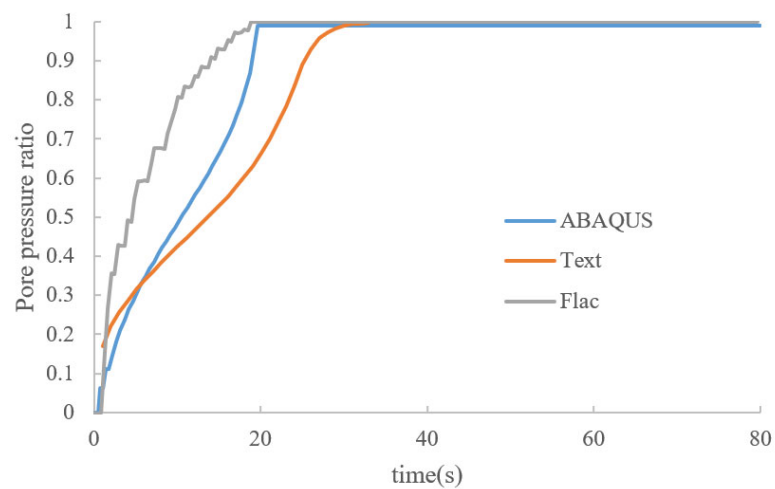


Figure 10. Dynamic triaxial tests 150kp0.21

It can be seen that under the conditions of high dynamic shear ratio and low confining pressure, FLAC3D and the subroutine can fit the test results quite well, and both show complete

liquefaction at the same time. The simulation and the test results indicate that with the increase of time, the pore pressure gradually increases until it reaches the initial effective confining pressure. At the same time, as the initial effective confining pressure increases, the trend of the pore pressure image under the same dynamic shear ratio is the same. This is consistent with the conclusion of the paper.

Overall, the fitting effect and the image curve of the subroutine are better than those of FLAC3D. There are several reasons for this result: On the one hand, the liquefaction calculation of the subroutine directly determines it by inputting sine strain into the subroutine, while FLAC3D determines it based on the sine strain on the model. On the other hand, the reduction method of the soil in the subroutine is different from that of FLAC3D, resulting in different trajectories of the soil liquefaction image. From the perspective of confining pressure, as the confining pressure increases, the simulation of the test by the subroutine and FLAC3D gradually increases the error. From the perspective of dynamic shear ratio, the larger the dynamic shear ratio, the smaller the error in the numerical simulation. The paper believes that the following two reasons lead to this result: 1. FLAC3D and the subroutine do not reflect the characteristic of pore pressure dissipation. 2. There is an error between converting fixed stress into fixed strain for participation in calculation and directly using fixed stress for participation in calculation. As the degree of liquefaction deepens, the relevant modulus of the soil will be reduced accordingly, resulting in a change in strain. And the smaller the initial effective confining pressure is and the larger the dynamic shear ratio is, the faster the soil liquefies and the smaller the influence of the strain error.

4. Conclusion

By combining the modified Hardin-Drnevich soil constitutive model with the modified Martin pore pressure increment model by Byrne, an ABAQUS subroutine based on the modified Hardin-Drnevich soil constitutive model and the Byrne pore pressure increment model was implemented. Using the ABAQUS subroutine and FLAC3D separately, numerical simulations were conducted for the dynamic triaxial test of saturated fine sand in Nanjing. The following conclusions were obtained:

1. In the cases of high dynamic shear ratio and low confining pressure, the subroutines and FLAC3D have fitting to the test results approximately. Both reflect the characteristic that with the increase of time, the pore pressure gradually increases until it reaches the initial effective confining pressure, and the same dynamic shear ratio, the pore pressure ratio image trend is the same as that of the initial effective confining pressure increasing.
2. It was proved that in the dynamic triaxial liquefaction test, the fitting effect and image curve of the subroutines are better than those of FLAC3D. This is due to the different soil reduction methods and sine wave input methods of the subroutines.
3. The reasons for the increase in error between FLAC3D and the ABAQUS subroutine as the confining pressure rises and the dynamic shear ratio decreases were discussed. This paper believes that it is a related problem caused by the fact that neither FLAC3D nor the subroutines can reflect the characteristics of pore pressure dissipation, and the fixed stress is converted into fixed strain for calculation and directly using the fixed stress for calculation. There is an error between them.

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